STUDY OF THE DYNAMICS OF ECONOMETRIC SYSTEMS BY LINEAR MULTIVARIABLE MODELS APPROACH

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Abstract

The linear relationship approach for econometric models is usually preferred to explore the socio-economical states and their dynamics, but physically one need equilibrium arguments to make them meaningful and quantitatively prescriptive. In this study we evaluate the linear approach for the fraction of votes casted and alternative Okun's law for the employment in Kosovo and Albania, to analyse the phase of dynamics of particular states. Linear mono variable models fail to describe the trend of some operands, and their respective elasticises were found not constant over time but including more macro parameters good fitting results are found. The fraction of votes of incumbent coalition as response of the most important macro indicators do shows instabilities suggesting the strong presence of other variable of no economic nature. Employment and unemployment in Albania and Kosovo are found in complexes dynamic by empiric linear models but with some extend they can be reduced to a linear form under few variable as GDP growth, CPI or if time is used as parameter. These results are considered as strong indicators that the socioeconomic states are in a transitive phase far from equilibrium with rich dynamic.

Key words: linear econometric models, fraction of votes, employment rate, equilibrium

Introduction

Econometrists like natural scientists are happy with linear equation even they don't believe that socio-economic parameters (called macro) behave so good as linear formulas do. Usually physicist will accept steady state ore equilibrium if some derivative of variables remain constant over time and therefore the dynamics of the system could be possible. With some attention we can use alternatively those views to explore the nature of the socioeconomic state. An old idea that prices can be expressed linearly as function of some input parameters involved in production process has been applied to explain the fraction of votes firstly by Cramer (1970) and others scholars. The underlying assumption on those models seems to be a quasi thermodynamic equilibrium approach on the sense that involved elasticities like capacities remain constant over time and there are no locally dynamics on the system. For the two party system has been proposed the formula

$$v_t^j = v_{oj} \pm \left(s_0 + \sum_{q=1}^{Q} s_q x_q + \Delta_t(x_q) \right) + y_t,$$ where v is the fraction of votes casted by the subject, x are indicators as GDP, Inflation, Employment etc, s are coefficients and y the term of error or noise. Fair gave a shortened form considering only a group of relevant indicators or changes of indicators as

$$V_p = c_p + c_G * i_G - c_I * i_I - c_p * i_P + c_C * i_C - c_{Nd} * k_{Nd} + c_{Lm} * n_{Lm} - c_{Lk} * n_{Lk} + y$$ focusing therefore on *GDP, Inflation, Good News/Bad News, and Number of Mandates* for incumbent parties. Here C_p is a constant, free of indicator dependency that can be assimilated on militant fraction. Another linear approach considered here is found on so called Okun's law that lies the employment rate with GDP growth $E = r(\Delta GDP) + s$ or $Log(E) = r(\Delta Log(GDP)) + s$. Again the rate of influence of some indicator will prescribe some behaviour on the system if one have been accepted the assumption of equilibrium. If dynamic process took place locally or globally, very complex processes tends to relax the socioeconomic system on a long time approach strengthening so far the correlation, like the diffusion in physics push the system toward steady state. But during this intermediate phase the indicators itself seems to be not strictly defined as evolving by times and so does the respective elasticities. Involving more parameters makes the linearization more meaningful and descriptive. Another difficulty of linear equation is related with particles migration that causes external in/out flows to destroy the equilibrium attempted by internal processes. We use those elements to identify the level of complexity of system considered and the dynamics of the overall state.

Dynamics of fractions of votes in elections by the linear approach.

We consider the Fair formula making some adoption to better fit our reality. First we use the coalition of parties as incumbent party on the original model as for all period of consideration the government has been made up of many parties that were together on even on the next election. Second we see that the corruption and fight against it has been very important issue on election. Employment strategies and unemployment rates where on the top list of engagement or critiques for all campaigns. Therefore we involve on Fair formula those two terms. Another difference compared with Fair idea is the fact that we don't use the quarterly indicators as they are very questionable: claims and reviews accompanied the statistics on those issues and we decide to take data from international institutional that report the data annually. Finally we had a subjective intervention on the formula in the case of *Good and Bad News* as there is a lot of discussion during years for what is *News*. For example even some specialist and opinion don't agree that the Gerdec tragedy was considered by peoples as bad news for incumbent party on 2009, and it was impossible to bring an opinion from enquires, we assign it as true-it was a *Bad News* for incumbent party!. According to the Fair approach coefficient should reflect the natural trend so we assign the natural sign to every indicator parameters that is if GDP has a positive growth the related incumbent party coefficient should be positive and so on. By this convention the solution should be positive. Having C=[*indicators*] and F=[*fraction of votes*] the linear form CA=F solved mathematically didn't meet this criteria. As seen the solution is not positive as by condition or says no logically "*no*

militant fraction is on the game " and so on. Calculation are performed for two cases "*all election considered political*" and " *only parliamentary election to be political*". No remarkable deference is seen. Really we have some indication for the linear model by correlation coefficient that one by one tells that only growth of GDP and the Mandates consumed are correlated (anti correlated) in the case of incumbent coalition as on Tab 1

GDP(growth)	Inflation	Unemployment	Corrupts	G.N	BN	Mandates
0.578144	-0.28423	0.369927	-0.08419	-0.41934	-0.40535	-0.60252

Tab1.

Not doubting at this step on the model, two explanatory arguments remain, the one of incorrect indicators and the other incorrect problem definition. We choose this last charging for it the dynamic of system; we can't guess that coefficients are to be time independent. Roughly speaking this situation will make the linear model inapplicable but we will keep it for some. Therefore a more acceptable strategy is to perform a dynamic approach following the idea that every indicator must affect that results logically as our constraints says, so an admissible idea for coefficients will be the approximates ones that meet sufficiently criteria. The difference from the real values is interpreted as deviation of the model from the reality overlapped with an error term, rather than pure noise. We start from some initial value and run the procedure until a plausible difference from real fraction is found under the constraint "all coefficients nonzero". Another limitation for natural solution is from the condition that the free coefficient can not be grater than 0.5, that is the model to be logic.

		Incumbent Coalition			Opposition		
		2001	2005	2009	2001	2005	2009
C_{inf}	Economica Indicators	7.4815	2.4018	0.3625	9.9694	6.5045	1.9814
C_{Gdp}		8.8771	9.6216	9.5769	2.565	9.3232	2.6722
C_{pp}		0.2616	1.9764	1.9405	0.0901	0.7386	0.3659
C_{lk}	Social Indicators	3.5379	0.2135	1.9393	0.2232	3.0676	0.326
C_{lm}		5.4054	0.8548	9.9956	9.7786	2.6819	2.8476
C_{korr}		8.3814	9.5668	1.2886	7.1798	6.9324	9.8967
C_{mand}		4.9293	5.4582	0.673	4.133	3.5771	6.6757
C_0		45.641	43.7568	40.0508	40.1365	45.1542	35.814
Dif		0.0046	0.0073	2.48E-04	5.61E-04	6.01E-04	0.0072

Tab2. Linearization using optimization under several constraints imposed.

Here we find that crude change from an election to the other is registered. From this point of view no forecast is possible the dynamic is so intensive and no relaxation even

partially is demonstrated. Nevertheless, an estimation procedure for forecasting can be introduced following the linearization idea, using the full range of elections. We added subjectively the electoral performance of subjects and the rank of parties on coalition so the redistribution of votes will be made according it. For election of 2009 the error was at 0.6% using second order polynomial extrapolation but there is no relevant aspect to be mentioned herein. Assuming all election as political and with some attention we can extrapolate to find that incumbent party will loose an extra 1.1% of popular votes on the election of 2013 with an error of 0.1%. The biggest contributors should be the decrease of militant fraction and an extra loose caused from the idea of time for change. But overall electoral state is clearly out of equilibrium and the linear dependencies are only locally meaningful. Therefore the political strategies could not be effective if based on realisations indicators as the behavioural reaction of the public is not proportional. It is not the place here to analyse the causes destroying the plausible and expected rationality. Physically we can say that this is caused primordially by the non equilibrium phenomena, which involve complex behaviour and interaction. So this seems to be a collective effect rather individual un-stabilised electoral culture. This last aspect might not differ apparently from other systems (country), but the system as whole does much as recently showed by network simulation[15]. Second the crude coefficient changes indicate that the system is not closed or diffusion based, interior migration and reconfiguration or displacement for another nature will be present, hiding the relaxing expected tendency. Thirdly, being the system far form steady state, the indicators measurement itself suffers form high error value. As long as all those phenomena will affect the system, linear approach continue to be globally not possible, but the distance form it can be a good estimation of complex dynamics of the system and can bee used as an alternative knowledge for reliability of methods for some indicator estimation too.

Linear model for employment in Kosovo

Another highly dynamic environment is expected to be the employment on the Kosovo Republic. There are many specific to be explained herein so we assume them as "publically known", as new state, transition economy high migration rate, dynamic population etc.   We consider the linear empiric formula derived from Okun's Law $E = \Gamma (\Delta GDP) + S$ or $Log(E) = \Gamma (\Delta Log(GDP)) + S$. Next we made a slight modification on the formula taking $E = \Gamma (\Delta GDP) + S (x)$ where x is another variable or group of variables, therefore residuals may not fit the standard error   distribution. But again the small number of point do not permit us for quantitative analyze, making overall study an qualitative view on the system. Nevertheless, statistics and the culture of facts seems to be stabilised in Kosovo, so better realistic results are expected. We focus our analyse on relationship among employment/unemployment rate, GDP and GDP growth, migration rate and the level of wages. The most characteristic parameter for this filed, Gross National Product is not available and we miss this check. Here e limit our self on the dependencies one by one using variables or their logarithm. The specialised enquiry from references [] has registered indicators and we use them to calculate the correlation coefficient. The indicators found on IMF and WB do show a similar picture. We see that a slight anticorrelation is found for GDP growth and unemployment rate. Testing a linear

fit for variables and their logarithm we don't find good agreement with Okun's law. The small number of data registered suggested to us a qualitative view.

Correlation coefficinets

Indicators	Fractions			Numerical value		
	Total	Female	Males	Total	Females	Males
GDP	0.38	0.03	**0.77**	**-0.83**	**0.58**	-0.28
GDP_Rr	**-0.62**	**-0.75**	-0.42	**-0.68**	0.23	-0.06
GDP(Nat.Curr	0.18	-0.14	0.61	**-0.84**	0.39	-0.46
CPI	0.01	-0.20	0.38	**-0.79**	0.07	**-0.64**
CPI(Index)	**-0.77**	**-0.83**	-0.42	-0.30	-0.18	**-0.52**
ImpTotal_Change	**0.65**	**0.63**	0.36	-0.37	-0.06	0.04
ExpTot_change)	0.13	0.47	-0.16	**0.66**	**-0.55**	-0.05

Tab2.

The relationship of Employment rate and GDP growth is not suitable by linear approach
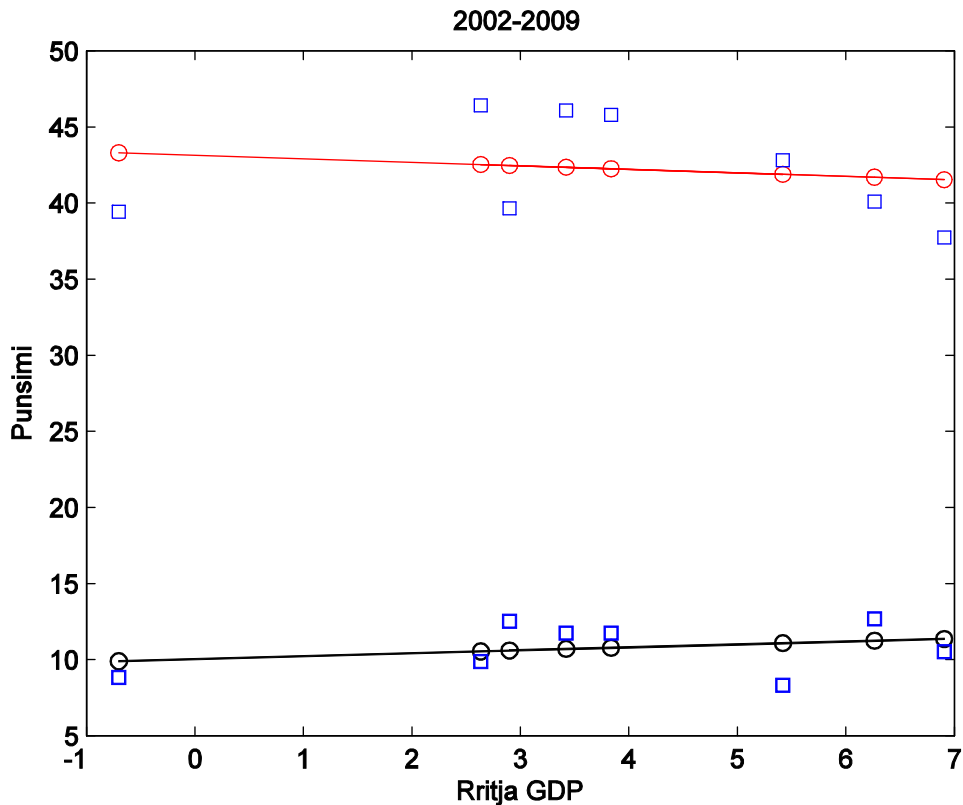

Fig1. Linear approach of Employment and GDP growth.
Intertsingly the amplyment of males is anticorreleted with GDP growth while for females is positive (vectors of solution are vm=[-0.2309   43.1376   82.4092] dhe vf =[0.1921   10.0399   17.8504].
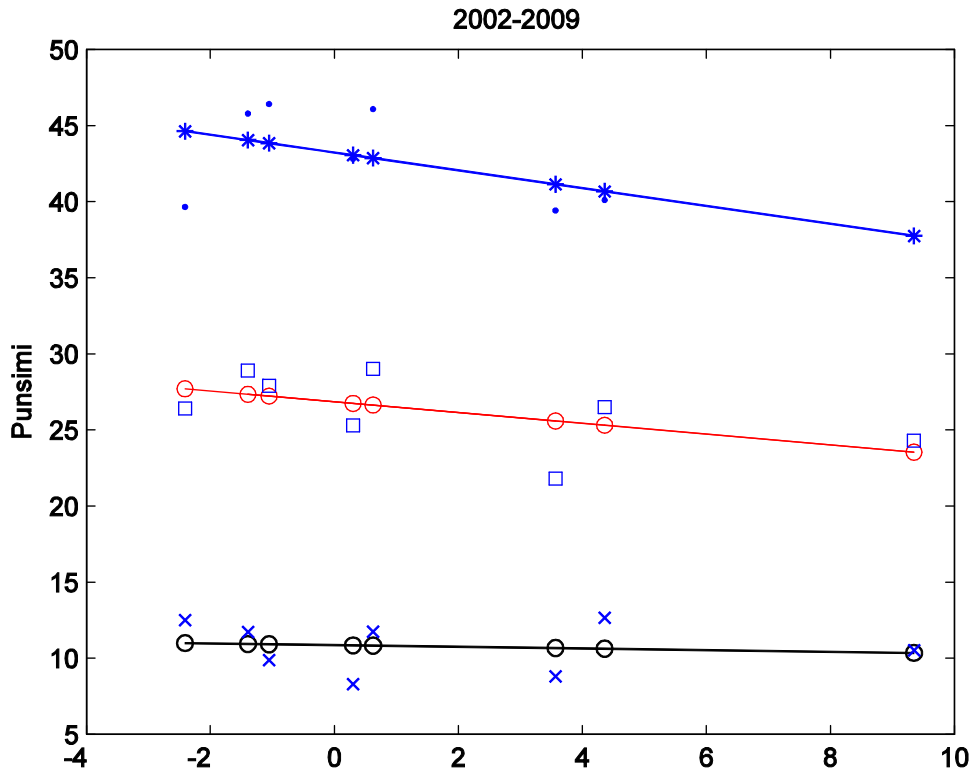
2002-2009

Fig4. CPI index and Unemployment rates

Noisy linear approach are found for realtionshim unemployment and CPI index. (fg4).
We've tested a polynomial of 3^d grade ax^3+bx^2+cx+d that gave a very good fit with
a=57.1079 b=-214.1720 c=266.6337 d=-106.8949 . By nature it is only a mathematical
approximated evaluation as no such model has been considered. We expected such
behaviour as the economy is complex and dynamic. Therefore a multivariable linear
approach was tested successively. Considering variables GDP growth, the Import
Volume change and export volume change we see that the curves is better fitted and this
behaviour could be at least meaningfully. The change in import volume is an hidden
indicator that demonstrate the ability of the country to trade goods and a possible
increase of services at least absorbing seasonal labour forces. The exports will told us the
quantity of production that use labour forces continuously therefore reflecting the
stability of amount of work force used in. Writing
$P = a + a_1 * GDP + a_2 * CPI + a_3 * IMP$ we found [a,a1,a2,a3] = [24.3216   0.6655
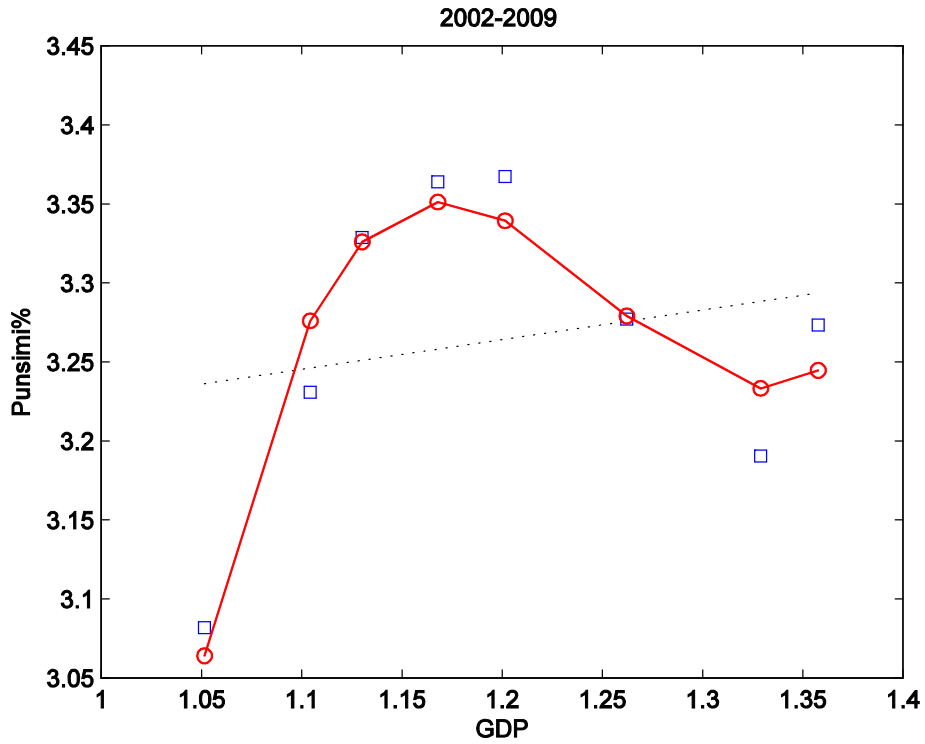0.0047   -0.5335] with lower value of residuals.

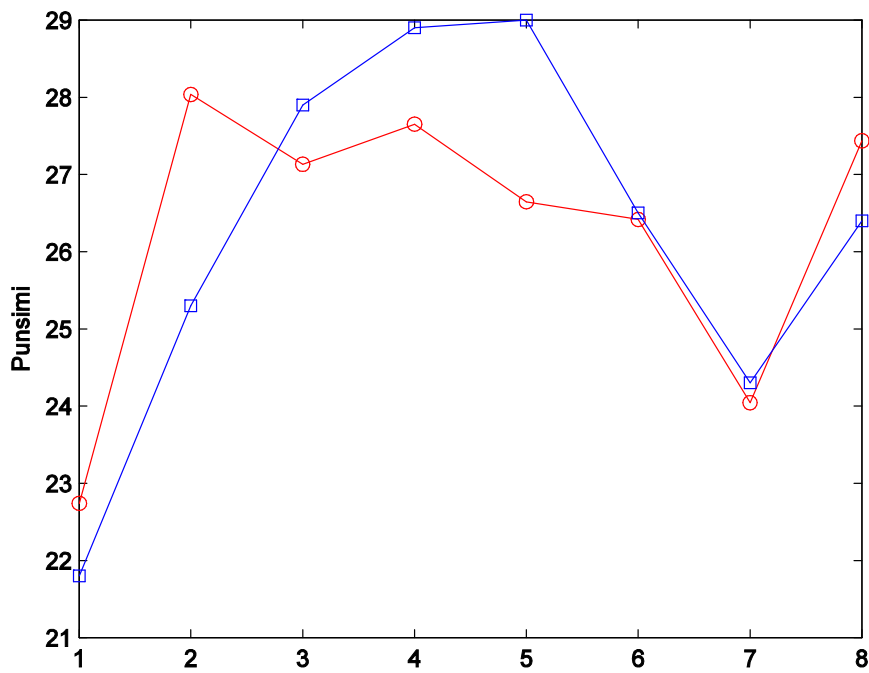Fig2. Linear and polynomial approach for Employment-GDP relationship


Fig7. Three variable linear approach

Again finding of this paragraph indicate the complexity of structure of the society and economy an specific transition as Kosovo case could bee.

The unemployment in Albania.

This is questionable parameters as there is no stable methodical way to determinate it undoubted.   According Okun's law scholars change in growth rate will affect directly the change on emloymetn/unemployment

2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	[01-11]	07-11]
3000	4500	4500	4900	5300	#####	#####	#####	#####	#####	#####	0.45	-0.54
7.5	5	7	5.6	5.5	5.00	6.00	6.10	3.30	3.50	2.00	0.65	0.25
1	6	2.4	3.2	2.4	2.50	2.90	3.40	2.20	3.60	3.50	-0.16	-0.40
0.31	0.34	0.34	0.43	0.55	0.65	0.76	1.35	1.05	1.55	1.95	-0.97	-0.63
1	1.5	1.5	1.76	2.08	2.47	2.90	4.90	4.26	4.59	5.08	-0.86	-0.87
1	0.78	0.78	1.41	1.41	1.55	1.55	1.55	1.55	1.55	2.81	-0.95	-0.22
16	17	17	15.8	14.8	14.30	13.80	12.50	12.80	13.50	13.30		

It is clearly that even here the linear approach analyse do show controversial behaviour so the relationship should be more complex, that is many variables involved. The linear fits are not so good. We extend the Okun's form to other parameters where we thing on Inflation, the export-import balance. The result looks somewhat good as in fig 5
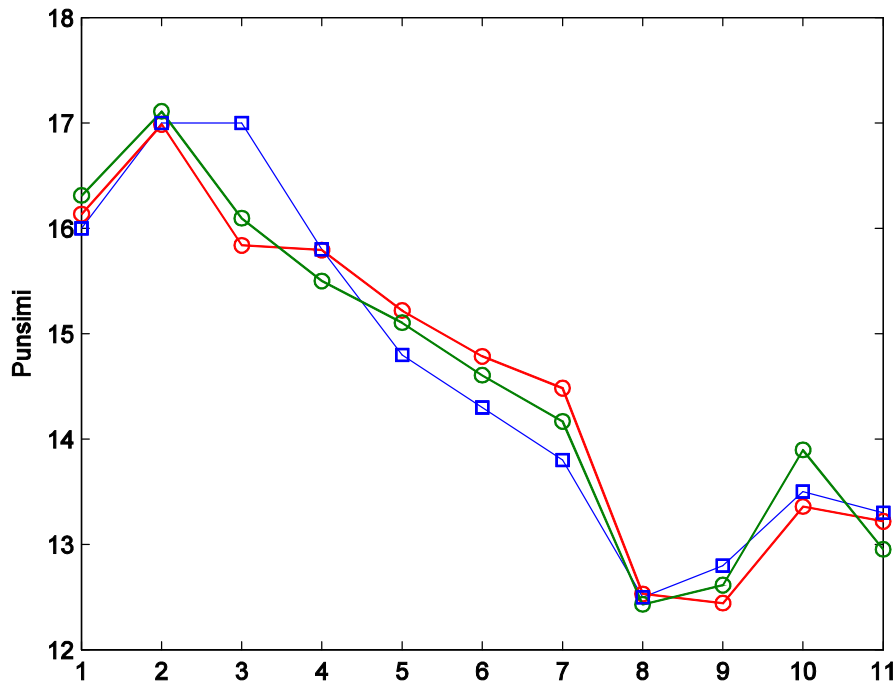
Parameters are [16.4511   0.0500   0.3472   1.9221   -1.6328], so the unemployment as reported is anti correlated only with volume of imports. Involving the public debt on the analyse we find again a good fit, better than the recent one (Fig5, line green). The functional form has coefficients for [GDP_G CPI V_Exp V_Imp Debt] as the vector [18.6763   -0.0816   0.2365   3.1626   -1.8571   -1.1108]. The last one resist to be negative again and the influence of public debt is negative on employment. This will be explained that place of jobs were open as of unbalanced contribution of public investment. During the period 2010-2011 the effect of involving the public debt seems to rise the difference of real data from the last modelled one.

Improved Okun's law.

Dealing with linear analyse we are aware that models can fail or data could have been collected un-appropriately. On the case study of employment/unemployment for Kosovo and Albania we suppose that another equation can be adopted to better explain the trends. We consider here the midiefied Okun's law on the form

$$u_t = u_0 + a \log\left[\frac{GDP\_G_t}{GDP\_G_0}\right] + b(t - t_0) + c$$ proposed on reference [7].   For Albania we

find a good approach by this model
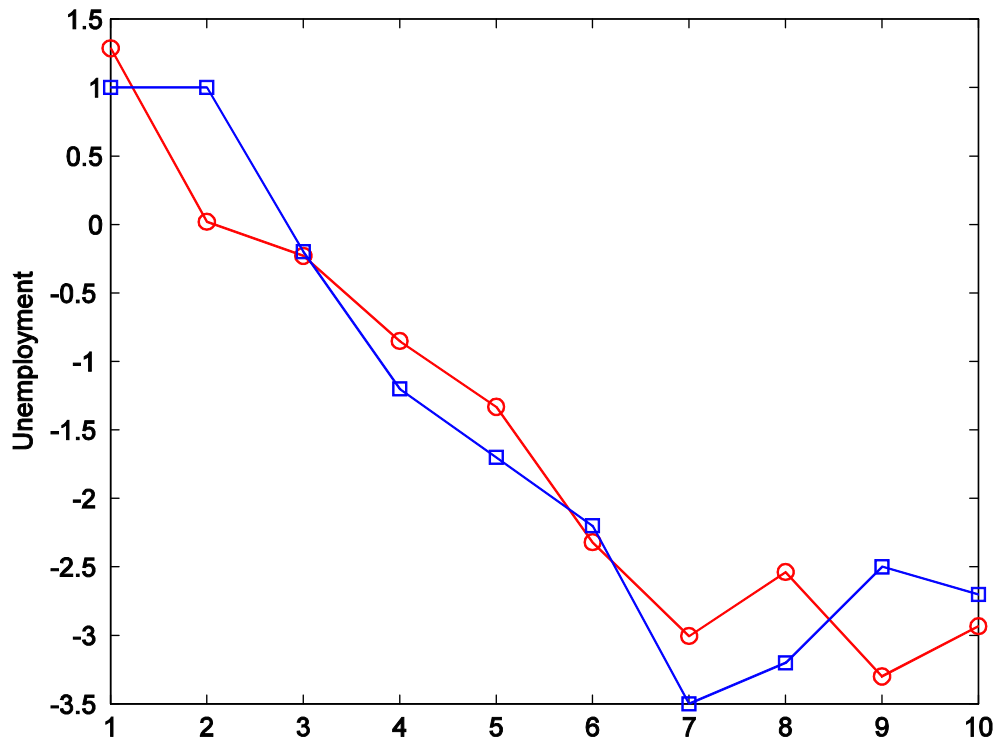

Fig6. Okuns'law modified-Albania

The parameters were found [a b c]=[1.2020   -4.2032   -0.6548]. Residuals are relatively small 2.6971 ensuring that a good approach is achieved. The same analysis for Kosovo gives a very good approach per the unemployment rate. The coefficient are found –[a b c]=[49.7101   -0.0261   58.9714]. The multi parametric approach tells for a complex relaxation tendency for the two systems considered if indicators estimation and their methodical are assumed to be credible enough. Nevertheless, the complex dynamics that do not permit a linear approach on our model is much better explained by a multivariable linear function and again more exact solution is found using modified Okuns law.
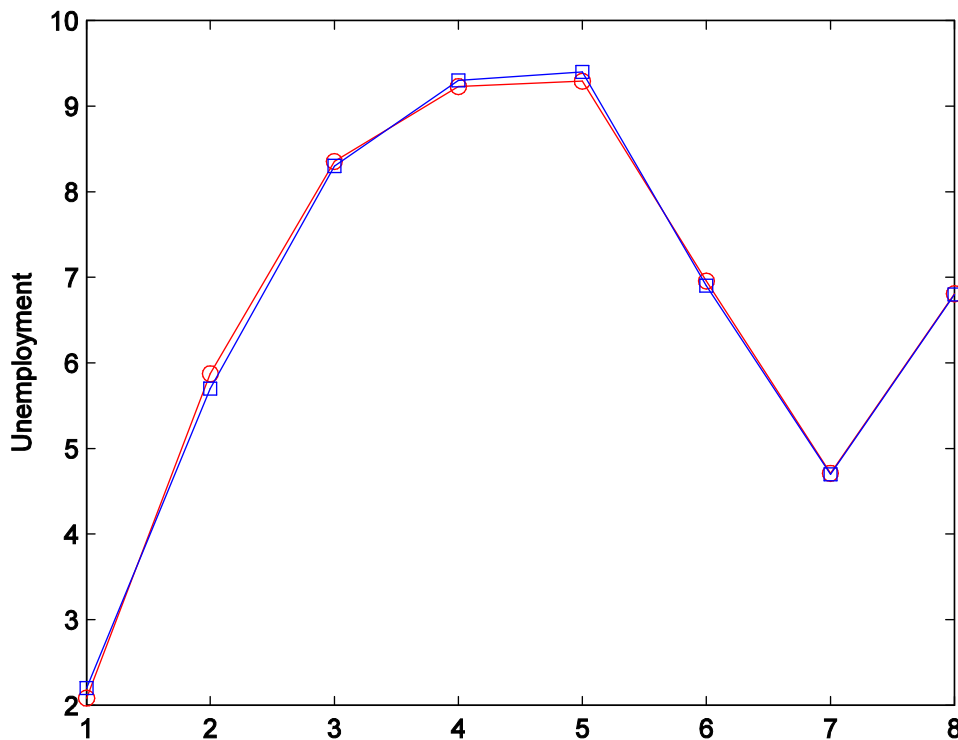

Fig10. Modified Okun's Law for unemployment rate for Kosova. By red line and marks, fitted curves, blue line, the real data point.

Conclusion

The study of linear forms for system with economy or society in transition reveal the importance of econometric linear approach as an multipurpose mean for the study. Even the linear fitting seems to be not a good approach, it give information of different aspects. Firstly we see that correlation as indicator for linear dependencies could be read as tendency or complexity of the system and relationship of variables. If illogic correlation is found comparing data series, another variable or phenomena is probably perturbing it. When transitive society or economy is considered we see that strong deviation form

standard behaviour on linear approach will take place. But selecting the natural variables affecting the state we consider, the linear dependency is restored and better fitted. The second remark on this view reveals the importance of accurate statistics and methodologies for indicator measuring. Considering the election result in Albania e see that linear approach analyse result on the evidence of intensive dynamic on the system. Applying linear models as Okun's law we see that the dynamic is present on Kosovo and Albania. The differences form expected behaviour are seen as indicators of complexity on both senses. Systems are out of equilibrium and indicators could not tell explicitly the behaviour, they could not be accurately known or descriptive, and multi dimensional interactions are still present. Transport phenomena, flows and migration may override diffusion causing the system to react unnaturally. Using multi parametric approach the hidden tendencies could be met on light and short term forecasting might bee possible to approximate the future behaviour.

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