ARITHMETIC OPERATIONS OF GENERALIZED TRAPEZOIDAL FUZZY NUMBER BAZED ON VERTEX METHOD

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ABSTRACT

Most of the mathematical problems are characterized as complex process for which complete information is not always available. To handle this, the problems need to be set up with the approximately available data. To make this possible Zadeh introduced fuzzy set theory.Fuzzy set theory is a mathematical method used to characterise and quantify uncertainty and imprecision in date and functional relationships. In recent years this subject has become an interesting branch of pure and applied sciences.To late we have the theory and applications of Generalized Fuzzy Number (GFN)also the function principle which could be used as the fuzzy numbers arithmetic operations between Generalized Fuzzy Numbers. . Recently Generalized Fuzzy Number has also used in many fields such as risk analysis, similarity measure, reliability etc. The difference between the arithmetic operations on generalized fuzzy numbers and the traditional fuzzy numbers is that the former can deal with both non-normalized and normalized fuzzy numbers but the later with normalized fuzzy numbers. In this paper, we have discussed four arithmetic operations (addition, subtraction, multiplication, division) for two Generalized Trapezoidal Fuzzy Number (GTrFNs) based on vertex method.This method is more useful than extension principle method and interval method in the case of expressions with two or more arithmetic operations. In section-2 we have compared this method based on an example. . In section-3 based on these operations we have solved some elementary problems of mensuration and have calculated required approximated values. . Further GTrFN can be used in various problems of mathematical sciences.

Keywords.Arithmetic,Operation,Trapezoidal Fuzzy Number(TrFN), Generalized Fuzzy Number (GFN), Generalized Trapezoidal Fuzzy Number (GTrFNs).

1. Mathematical Preliminaries

Definition 1.1: Fuzzy Set: A fuzzy set Ã in a universe of discourse X is defined as the following set of pairs Ã={(χ,$\mu_{\tilde{A}(\chi)}$):χ∈X} Here$\mu_{\tilde{A}}$: X[0,1] is a mapping called the membership value of x∈X in a fuzzy setÃ.

Definition 1.2: Fuzzy Number: A fuzzy number is an extension of a regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. Thus a fuzzy number is a convex and normal fuzzy set. If Ã is a fuzzy number then Ã is a fuzzy convex set and if then is non decreasing for $\chi \leq \chi_0$ and non increasing for $\chi \geq \chi_0$.

Definition 1.3: Trapezoidal Fuzzy Number: A Trapezoidal fuzzy number (TrFN) denoted by Ã is defined as (a_1,a_2,a_3,a_4)where the membership function

$$\mu_{\tilde{A}}(\chi)=\begin{cases} 0, & x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\ 0, & x \geq a_4 \end{cases}$$

Or, $\mu_{\tilde{A}}(x) = \max(\min(\frac{x-a_1}{a^2-a^1},1,\frac{a_4-x}{a_4-a_3}),0)$.

Definition 1.4: Generalized Fuzzy number (GFN): A fuzzy set Ã=(a_1,a_2,a_3,a_4,w);, defined on the universal set of real numbers *R*, is said to be generalized fuzzy number if its membership function has the following cha, racteristics:

(1)$\mu_{\tilde{A}(\chi)}$: R→ [0, 1]is continuous
(2) $\mu_{\tilde{A}(\chi)}$ =0 for all χ∈ (-∞,a_1]U [a_4,∞)
(3) $\mu_{\tilde{A}(\chi)}$s strictly increasing on [a_1 ,a_2] and strictly decreasing on [a_3,a_4].
(4) $\mu_{\tilde{A}(\chi)}$ =w for all χ ∈ [a_2,a_3], where 0 < w≤ 1 .

Definition1.5: Generalized Trapezoidal Fuzzy number (GTrFN): A Generalized Fuzzy Number Ã=(a_1,a_2,a_3,a_4,w), is called a Generalized Trapezoidal Fuzzy Number if its membership

$$\mu_{\tilde{A}}(\chi)=\begin{cases} 0, & x \leq a_1 \\ w\frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ w & a_2 \leq x \leq a_3 \\ w\frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\ 0, & x \geq a_4 \end{cases}$$

Or, $\mu_{\tilde{A}}(x) = \max(\min(w\frac{x-a_1}{a^2-a^1}, w, w\frac{a_4-x}{a_4-a_3}), 0)$.

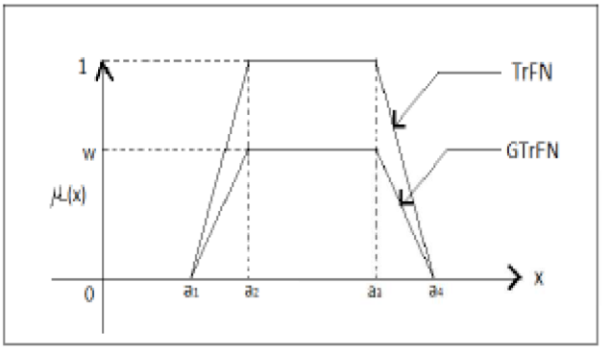

Fig 1 : Comparison between membership function of TrFN and GTrFN.

2. Arithmetic operations of GTrFNs

In this section we discuss four operations (addition, subtraction, multiplication, division) for two generalized trapezoidal fuzzy numbers based on vertex method.

Let $\tilde{A} = (a_1, a_2, a_3, a_4, w_1)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, w_2)$ be two positive generalized trapezoidal fuzzy numbers and their membership functions are :

$\mu_{\tilde{A}}(x) = \max(\min(w_1\frac{x-a_1}{a^2-a^1}, w_1, w_1\frac{a_4-x}{a_4-a_3}), 0)$.

$\mu_{\tilde{B}}(x) = \max(\min(w_2\frac{y-b_1}{b_2-b_1}, w_2, w_2\frac{b_4-y}{b_4-b_3}), 0)$.

and their $\propto$ -cuts be

$A_\propto = [A_1(\propto), A_2(\propto)] = [a_1 + \frac{\propto}{w_1}(a_2 - a_1), a_4 - \frac{\propto}{w_1}(a_4 - a_3)]$, $\forall \propto \in [0, w_1]$, $0 < w_1 \leq 1$

$B_\propto = [B_1(\propto), B_2(\propto)] = [b_1 + \frac{\propto}{w_2}(b_2 - b_1), b_4 - \frac{\propto}{w_2}(b_4 - b_3)]$, $\forall \propto \in [0, w_2]$, $0 < w_2 \leq 1$

2.1 Addition of two GTrFNs

Let $\tilde{C} = f(\tilde{A}, \tilde{B}) = \tilde{A} + \tilde{B}$

Now the ordinate of the vertices are

$c_1 = (a_1 + \frac{\propto}{w}(a_2 - a_1), b_1 + \frac{\propto}{w}(b_2 - b_1))$, $c_2 = (a_1 + \frac{\propto}{w}(a_2 - a_1), b_4 - \frac{\propto}{w}(b_4 - b_3))$

$c_3 = (a_4 - \frac{\propto}{w}(a_4 - a_3), b_1 + \frac{\propto}{w}(b_2 - b_1))$, $c_4 = (a_4 - \frac{\propto}{w}(a_4 - a_3), b_4 - \frac{\propto}{w}(b_4 - b_3))$

and $f(c_1) = a_1 + b_1 + \frac{\propto}{w}(b_2 + a_2 - a_1 - b_1)$ $\quad f(c_2) = a_1 + b_4 + \frac{\propto}{w}(a_2 - b_4 - a_4 + b_3)$

$f(c_3) = a_4 + b_1 + \frac{\propto}{w}(b_2 - a_4 + a_3 - b_1)$ $\quad f(c_4) = a_4 + b_4 + \frac{\propto}{w}(b_3 + a_3 - a_4 - b_4)$

It can be shown that $f(c_1)<f(c_2)<f(c_3)<f(c_4)$:

So $Y=[\min(f(c_1),f(c_2),f(c_3),f(c_4)),\max(f(c_1),f(c_2),f(c_3),f(c_4))]$

$=[f(c_1), f(c_4)]$

$=[a_1+b_1+\frac{\propto}{w}(b_2+a_2-a_1-b_1), a_4+b_4+\frac{\propto}{w}(b_3+a_3-a_4-b_4)]$

[Note: $x\geq a_1+b_1+\frac{\propto}{w}(b_2+a_2-a_1-b_1)\Rightarrow w\frac{x-(a_1+b_1)}{(a_2+b_2)-(a_1+b_1)}\geq\propto\Rightarrow\mu_{\tilde{C}}^{L}(x)\geq\propto$

$x\leq a_4+b_4-\frac{\propto}{w}(a_4+b_4-a_3-b_3)\Rightarrow w\frac{(a_4+b_4)-x}{(a_4+b_4)-(a_2+b_2)}\geq\propto\Rightarrow\mu_{\tilde{C}}^{R}(x)\geq\propto$]

The addition of two $\tilde{A}$, $\tilde{B}$ GTrFNs is another GTrFN $\tilde{C}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4,w)$with membership function given at equation and shown in Fig2

and $\mu_{\tilde{C}}(z)=\begin{cases} w\frac{z-a_1-b_1}{a_2+b_2-a_1-b_1} & if \quad a_1+b_1\leq z\leq a_2+b_2 \\ w & if \quad a_2+b_2\leq z\leq a_3+b_3 \\ w\frac{a_4+b_4-z}{a_4+b_4-a_3-b_3} & if \quad a_3+b_3\leq z\leq a_4+b_4 \\ 0 & otherwise \end{cases}$

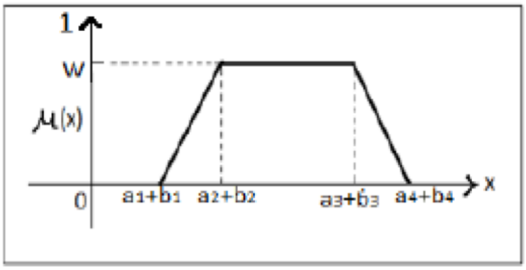

Fig 2:- Rough sketch of Membership function of $\tilde{A}(+)\tilde{B}$.

2.2 Scalar multiplication of a GTrFN .

Let $\tilde{C}=f(\tilde{A})=k\tilde{A}$

Now the ordinate of the vertices are

$c_1=(a_1+\frac{\propto}{w}(a_2-a_1))$, $c_2=(a_4-\frac{\propto}{w}(a_4-a_3))$

and $f(c_1)=k(a_1+\frac{\propto}{w}(a_2-a_1))$ $f(c_2)=k(a_4-\frac{\propto}{w}(a_4-a_3))$

Case1: When $k>0$, $f(c_1)<f(c_2)$:

So $Y=[\min(f(c_1),f(c_2)),\max(f(c_1),f(c_2))]=[f(c_1), f(c_2)]$

$=[k\,a_1+\frac{\propto}{w_1}k(a_2 - a_1)\,, k\,a_4 - \frac{\propto}{w}\,k\,(a_4 - a_3)]$

[Note : $x \geq k\,a_1+\frac{\propto}{w_1}k(a_2 - a_1) \Rightarrow w\frac{x\text{-}ka_1}{ka_2\text{-}ka_1} \geq \propto \Rightarrow \mu^L_{\tilde{C}}(x) \geq \propto \Rightarrow \mu^L_{\tilde{C}}(x) \geq \propto$

$x \leq k\,a_4 - \frac{\propto}{w}\,k\,(a_4 - a_3) \Rightarrow w\frac{ka_4\text{-}x}{ka_4\text{-}ka_3} \geq \propto \Rightarrow \mu^R_{\tilde{C}}(x) \geq \propto \Rightarrow \mu^R_{\tilde{C}}(x) \geq \propto]$

The positive scalar (k) multiplication of a GTrFN Ã is another GTrFN $\tilde{C}$=($ka_1, ka_2, ka_3, ka_4; w$)

with membership function given at given at equation and shown in Fig3.

$$\text{and } \mu_{\tilde{C}}(z) = \begin{cases} w\,\frac{z-ka_1}{ka_2\text{-}ka_1} & if \;\; k\,a_1 \leq z \leq ka_2 \\ w & if \;\; ka_2 \leq z \leq ka_3 \\ w\,\frac{ka_4\text{-}z}{ka_4\text{-}kb_3} & if \;\; ka_3 \leq z \leq ka_4 \\ 0 & otherwise \end{cases}$$

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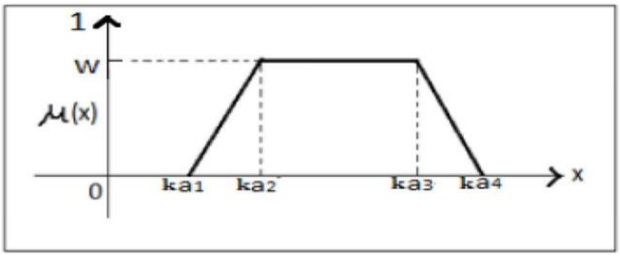

Fig 3 :- Rough sketch of Membership function of kÃ .

Case2: When k < 0 , $f(c_1) > f(c_2)$

So Y =[min($f(c_1), f(c_2)$),max ($f(c_1), f(c_2)$)]

=[$f(c_2), f(c_1)$]

[Note :

$x \geq k\,a_4 - \frac{\propto}{w}\,(ka_4 - ka_3) \Rightarrow w\frac{x\text{-}ka_4}{ka_4\text{-}ka_3} \geq \propto \Rightarrow \mu^L_{\tilde{C}}(x) \geq \propto \Rightarrow \mu^L_{\tilde{C}}(x) \geq \propto$

$x \leq k\,a_1+\frac{\propto}{w_1}(ka_2 - k\,a_1) \Rightarrow w\frac{ka_1\text{-}x}{ka_2\text{-}ka_1} \geq \propto \Rightarrow \mu^R_{\tilde{C}}(x) \geq \propto \Rightarrow \mu^R_{\tilde{C}}(x) \geq \propto]$

The negative scalar (k) multiplication of a GTrFN is another GTrFN $\tilde{C}$=($ka_4, ka_3, ka_2, ka_1; w$) with membership function given at equation an shown in Fig 4.

$$\text{and } \mu_{\tilde{C}}(z) = \begin{cases} w\,\frac{z-ka_4}{ka_4\text{-}ka_3} & if \;\; k\,a_3 \leq z \leq ka_4 \\ w & if \;\; ka_2 \leq z \leq ka_3 \\ w\,\frac{ka_1\text{-}z}{ka_2\text{-}kb_1} & if \;\; ka_1 \leq z \leq ka_2 \\ 0 & otherwise \end{cases}$$

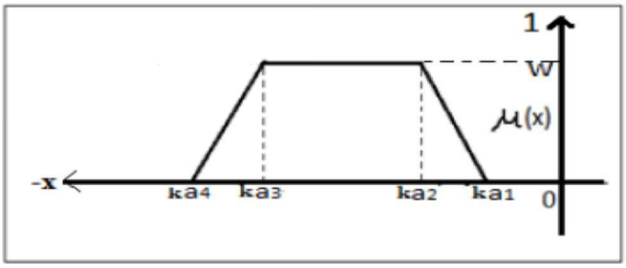

Fig 4:- Rough sketch of Membership function of kÃ.

2.3 Subtraction of two GTrFNs

Let $\tilde{C} = f(\tilde{A}, \tilde{B}) = \tilde{A} - \tilde{B}$

Now the ordinate of the vertices are:

$c_1 = (a_1 + \frac{\propto}{w}(a_2 - a_1), b_1 + \frac{\propto}{w}(b_2 - b_1))$, $c_2 = (a_1 + \frac{\propto}{w}(a_2 - a_1), b_4 - \frac{\propto}{w}(b_4 - b_3))$

$c_3 = (a_4 - \frac{\propto}{w}(a_4 - a_3), b_1 + \frac{\propto}{w}(b_2 - b_1))$, $c_4 = (a_4 - \frac{\propto}{w}(a_4 - a_3), b_4 - \frac{\propto}{w}(b_4 - b_3))$

and $f(c_1) = a_1 - b_1 + \frac{\propto}{w}(a_2 - b_2 - a_1 + b_1)$ $\quad f(c_2) = a_1 - b_4 + \frac{\propto}{w}(a_2 + b_4 - a_1 - b_3)$

$f(c_3) = a_4 - b_1 + \frac{\propto}{w}(b_1 - a_4 + a_3 - b_2)$ $\quad f(c_4) = a_4 - b_4 + \frac{\propto}{w}(b_4 + a_3 - a_4 - b_3)$

It can be shown that $f(c_2) < f(c_1) < f(c_4) < f(c_3)$:

So $Y = [\min(f(c_1), f(c_2), f(c_3), f(c_4)), \max(f(c_1), f(c_2), f(c_3), f(c_4))]$

$= [f(c_2), f(c_3)](b_1 - a_4 + a_3 - b_2)$

$= [a_1 - b_4 + \frac{\propto}{w}(a_2 + b_4 - a_1 - b_3), a_4 - b_1 + \frac{\propto}{w}(b_1 - a_4 + a_3 - b_2)]$

[Note:

$x \geq a_1 + b_1 + \frac{\propto}{w}(b_2 + a_2 - a_1 - b_1) \Longrightarrow w\frac{x - (a_1 + b_1)}{(a_2 + b_2) - (a_1 + b_1)} \geq \propto \Longrightarrow \mu^L_{\tilde{C}}(x) \geq \propto$

$x \leq a_4 + b_4 - \frac{\propto}{w}(a_4 + b_4 - a_3 - b_3) \Longrightarrow w\frac{(a_4 + b_4) - x}{(a_4 + b_4) - (a_2 + b_2)} \geq \propto \Longrightarrow \mu^R_{\tilde{C}}(x) \geq \propto$]

The subtraction of two $\tilde{A}$, $\tilde{B}$ GTrFNs is another GTrFN $\tilde{C} = (a_1 - b_4, a_2 + b_3, a_3 - b_2, a_4 - b_1, w)$ with membership function given at equation and shown in Fig 5.

$$\text{and } \mu_{\tilde{C}}(z) = \begin{cases} w\,\frac{z - (a_1 - b_4)}{(a_2 - b_3) - (a_1 - b_4)} & if \quad a_1 - b_4 \leq z \leq a_2 - b_3 \\ w & if \quad a_2 - b_3 \leq z \leq a_3 - b_2 \\ w\,\frac{(a_4 - b_1) - z}{(a_4 - b_1) - (a_3 - b_2)} & if \quad a_3 - b_2 \leq z \leq a_4 - b_1 \\ 0 & otherwise \end{cases}$$

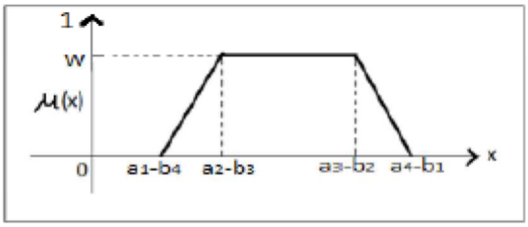

Fig 5:-Rough sketch of Membership function of $\tilde{A}(-)\tilde{B}$.

2.4 Multiplication of two GTrFNs

Let $\tilde{C} = f(\tilde{A}, \tilde{B}) = \tilde{A} \,.\, \tilde{B}$

Now the ordinate of the vertices are:

$c_1 = (a_1 + \frac{\propto}{w}(a_2 - a_1),\ b_1 + \frac{\propto}{w}(b_2 - b_1))$, $c_2 = (a_1 + \frac{\propto}{w}(a_2 - a_1),\ b_4 - \frac{\propto}{w}(b_4 - b_3))$

$c_3 = (a_4 - \frac{\propto}{w}(a_4 - a_3),\ b_1 + \frac{\propto}{w}(b_2 - b_1))$, $c_4 = (a_4 - \frac{\propto}{w}(a_4 - a_3),\ b_4 - \frac{\propto}{w}(b_4 - b_3))$

and $f(c_1) = \{a_1 + \frac{\propto}{w}(a_2 - a_1)\}\{b_1 + \frac{\propto}{w}(b_2 - b_1)\}$

$f(c_2) = \{a_1 + \frac{\propto}{w}(a_2 - a_1)\}\{b_4 - \frac{\propto}{w}(b_4 - b_3)\}$

$f(c_3) = \{a_4 - \frac{\propto}{w}(a_4 - a_3)\}\{b_1 + \frac{\propto}{w}(b_2 - b_1)\}$

$f(c_4) = \{a_4 - \frac{\propto}{w}(a_4 - a_3)\}\{b_4 - \frac{\propto}{w}(b_4 - b_3)\}$

It can be shown that $f(c_2) < f(c_1) < f(c_4) < f(c_3)$:

So $Y = [\min(f(c_1), f(c_2), f(c_3), f(c_4)), \max(f(c_1), f(c_2), f(c_3), f(c_4))]$

$= [f(c_2), f(c_4)]$

$= [\{a_1 + \frac{\propto}{w}(a_2 - a_1)\}\{b_1 + \frac{\propto}{w}(b_2 - b_1)\}, \{a_4 - \frac{\propto}{w}(a_4 - a_3)\}\{b_4 - \frac{\propto}{w}(b_4 - b_3)\}]$

[Note:Let $A_1 = (a_2 - a_1)(b_2 - b_1)$, $B_1 = \{a_1(b_2 - b_1) + b(a_2 - a_1)\}$, $C_1 = a_1 b_1$

and $A_1(\frac{\propto}{w})^2 + B_1\frac{\propto}{w} + C_1 - z \leq 0 \Rightarrow \frac{-B_1 - \sqrt{B_1^2 - 4A_1(C_1 - z)}}{2A_1} \leq \frac{\propto}{w} \leq \frac{-B_1 + \sqrt{B_1^2 - 4A_1(C_1 - z)}}{2A_1}$

and $\mu_{\tilde{C}}^{L}(z) = w\frac{-\{a_1(b_2 - b_1) + b_1(a_2 - a_1)\} + \sqrt{\{a_1(b_2 - b_1) + b_1(a_2 - a_1)\}^2 - 4(a_2 - a_1)(b_2 - b_1)(a_1 b_1 - z)}}{2(a_2 - a_1)(b_2 - b_1)} \geq \alpha$

$\frac{d\mu_{\tilde{C}}^{L}(x)}{dz} = \frac{w}{\sqrt{\{a_1(b_2 - b_1) + b(a_2 - a_1)\}^2 - 4(a_2 - a_1)(b_2 - b_1)(a_1 b_1 - z)}} > 0$

$\mu_{\tilde{C}}^{L}(z)$ is an increasing function in z.]

[N ote :Let $A_2=(a_4-a_3)(b_4-b_3)$, $B_2=-\{a_4(b_4-b_3)+b_4(a_4-a_3)\}$, $C_2=a_4b_1$

and $A_2(\frac{\alpha}{w})^2+B_2\frac{\alpha}{w}+C_2-z\leq 0 \Rightarrow \frac{\alpha}{w}\leq \frac{-B_2-\sqrt{B_2^2-4A_2(C_2-z)}}{2A_2}$ or $\frac{-B_2+\sqrt{B_2^2-4A_2(C_2-z)}}{2A_2}\geq\frac{\alpha}{w}$

and $\mu_{\tilde{C}}^{L}(z)= w\frac{-\{a_4(b_4-b_3)+b_4(a_4-a_3)\}+\sqrt{\{a_4(b_4-b_3)+b_4(a_4-a_3)\}^2-4(a_4-a_3)(b_4-b_3)(a_4b_4-z)}}{2(a_4-a_3)(b_4-b_3)}\geq\alpha$

$$\frac{d\mu_{\tilde{C}}^{L}(x)}{dz}=\frac{-w}{\sqrt{\{a_4(b_4-b_3)+b_4(a_4-a_3)\}^2-4(a_4-a_3)(b_4-b_3)(a_4b_4-z)}}<0$$

and $\mu_{\tilde{C}}^{L}(z)$ is a decreasing function in z.]

The multiplication of two GTrFNs $\tilde{A}$, $\tilde{B}$ is a generalized trapezoidal shaped fuzzy number $\tilde{C}\approx(a_1b_1,a_2b_2,a_3b_3,a_4b_4,w)$with membership function given at equation and shown in Fig 6.

and $$\mu_{\tilde{C}}(z)=\begin{cases} w\,\frac{-B_1+\sqrt{B_1^2-4A_1(C_1-z)}}{2A_1} & if\quad a_1b_1\leq z\leq a_2b_2 \\ w & if\quad a_2b_2\leq z\leq a_3b_3 \\ w\,\frac{-B_2-\sqrt{B_2^2-4A_2(C_2-z)}}{2A_2} & if\quad a_3b_3\leq z\leq a_4b_4 \\ 0 & otherwise \end{cases}$$

When $A_1=(a_2-a_1)(b_2-b_1)$, $B_1=\{a_1(b_2-b_1)+b_1(a_2-a_1)\}$, $C_1=a_1b_1$

$A_2=(a_4-a_3)(b_4-b_3)$, $B_2=-\{a_4(b_4-b_3)+b_4(a_4-a_3)\}$, $C_2=a_4b_4$

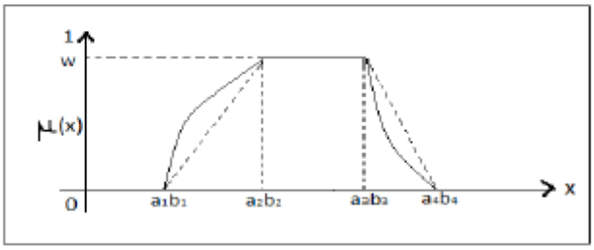

Fig 6 :-Rough sketch of Membership function of $\tilde{A}(.)\tilde{B}$.

2.5 Division of two GTrFNs

Let $\tilde{C}=f(\tilde{A},\tilde{B})=\frac{\tilde{A}}{\tilde{B}}$
Now the ordinate of the vertices are

$c_1=(a_1+\frac{\alpha}{w}(a_2-a_1), b_1+\frac{\alpha}{w}(b_2-b_1))$, $c_2=(a_1+\frac{\alpha}{w}(a_2-a_1), b_4-\frac{\alpha}{w}(b_4-b_3))$

$c_3=(a_4-\frac{\alpha}{w}(a_4-a_3), b_1+\frac{\alpha}{w}(b_2-b_1))$, $c_4=(a_4-\frac{\alpha}{w}(a_4-a_3), b_4-\frac{\alpha}{w}(b_4-b_3))$

and $f(c_1)=\frac{\{a_1+\frac{\alpha}{w}(a_2-a_1)\}}{\{b_1+\frac{\alpha}{w}(b_2-b_1)\}}$ $f(c_2)=\frac{\{a_1+\frac{\alpha}{w}(a_2-a_1)\}}{\{b_4-\frac{\alpha}{w}(b_4-b_3)\}}$ $f(c_3)=\frac{\{a_4-\frac{\alpha}{w}(a_4-a_3)\}}{\{b_1+\frac{\alpha}{w}(b_2-b_1)\}}$ $f(c_4)=\frac{\{a_4-\frac{\alpha}{w}(a_4-a_3)\}}{\{b_4-\frac{\alpha}{w}(b_4-b_3)\}}$

It can be shown that $f(c_2)<f(c_1)<f(c_4)<f(c_3)$:

So $Y = [\min(f(c_1), f(c_2), f(c_3), f(c_4)), \max(f(c_1), f(c_2), f(c_3), f(c_4))]$

$= [f(c_2), f(c_3)]$

$= \left[\frac{\{a_1 + \frac{\alpha}{w}(a_2 - a_1)\}}{\{b_4 - \frac{\alpha}{w}(b_4 - b_3)\}}, \frac{\{a_4 - \frac{\alpha}{w}(a_4 - a_3)\}}{\{b_1 + \frac{\alpha}{w}(b_2 - b_1)\}}\right]$

[N ote $\mu^L_{\tilde{C}}$ (z)= $w\frac{a_2b_4 - a_1b_2}{\{(a_2-a_1)+Z(b_3-b_2)\}^2} > 0$ for $a_2b_4 > a_1b_3$ and $\mu^L_{\tilde{C}}$ (z) is an increasing function in z.

$\frac{d\mu^R_{\tilde{C}}(z)}{dx} = w\frac{a_3b_1 - a_4b_2}{\{(a_3-a_2)+Z(b_2-b_1)\}^2} < 0$ for $a_3b_1 < a_4b_2$ and $\mu^R_{\tilde{C}}$ (z) is an increasing function in z.

Again $\mu^L_{\tilde{C}}(\frac{a_2}{b_3}) = \mu^R_{\tilde{C}}(\frac{a_3}{b_2}) = w$ and $\mu^L_{\tilde{C}}(\frac{a_1}{b_4}) = 0$, $\mu^R_{\tilde{C}}(\frac{a_4}{b_1}) = 0$

$\mu^L_{\tilde{C}}\left(\frac{\frac{a_1}{b_4}+\frac{a_2}{b_3}}{2}\right) = \frac{wb_4}{b_4+b_3} > \frac{w}{2}$ $[b_3 < b_4]$ and $\mu^R_{\tilde{C}}\left(\frac{\frac{a_3}{b_2}+\frac{a_4}{b_1}}{2}\right) = \frac{wb_1}{b_1+b_2} < \frac{w}{2}$ $[b_1 < b_2]$]

we get that the division of two GTrFNs $\tilde{A}$, $\tilde{B}$ is a generalized trapezoidal shaped fuzzy number $\tilde{C} \approx (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}, w)$ with membership function given at equation and shown in Fig 7

and $\mu_{\tilde{C}}(z) = \begin{cases} w\frac{zb_4 - a_1}{a_2 - a_1 + z(b_4 - b_3)} & if \quad \frac{a_1}{b_4} \le z \le \frac{a_2}{b_3} \\ w & if \quad \frac{a_2}{b_3} \le z \le \frac{a_3}{b_2} \\ w\frac{a_4 - zb_1}{a_4 - a_3 + z(b_2 - b_1)} & if \quad \frac{a_3}{b_2} \le z \le \frac{a_4}{b_1} \\ 0 & otherwise \end{cases}$

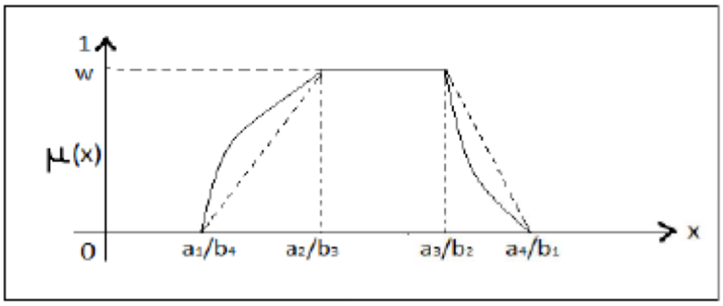

Fig7:- Rough sketch of Membership function of $\tilde{A}(:)\tilde{B}$.

3 Vertex method based on an example

Determine. $Y = f(X_1, X_2, X_3) = X_1X_2 + X_3$.
Given
$X_1 = [2,3]$, $X_2 = [4,5]$, $X_3 = [6,7]$
The ordinate of vertices are
$c_1=(2,4,6)$, $c_2=(2,4,7)$, $c_3=(2,5,6)$, $c_4=(2,5,7)$, $c_5=(3,4,6)$, $c_6=(3,4,7)$, $c_7=(3,5,6)$, $c_8=(3,5,7)$

From those ordinates, we obtain
$f(c_1)$=2.4+6=14 , $f(c_2)$=2.4+7=15 , $f(c_3)$=2.5+6=16 , $f(c_4)$=2.5+7=17 ,
$f(c_5)$=3.4+6=18 , $f(c_6)$=3.4+7=19 , $f(c_7)$=3.5+6=21 , $f(c_8)$=3.5+7=22
Then Y =[min(14,15,16,17,18,19,21,22) , max(14,15,16,17,18,19,21,22)]=[14,22]

Conclusion and future work

In this paper, we have worked on GTrFN. We have described four operations for two GTrFNs based on vertex method and an example.. Further GTrFN can be used in various problems of engineering and mathematical sciences.

References

L. A. Zadeh, Fuzzy sets, Information and Control, 8(1965), 338-353.

Chen, S. H. 1985. Operations on fuzzy numbers with function principal. Tamkang Journal of Management Sciences,6: 13-25.

Hsieh, C. H. and Chen, S. H. 1999.Similarity of generalized fuzzy numbers with graded mean integration representation. In: Proceedings of the Eighth International Fuzzy System Association World Congress, Taipei, Taiwan, Republic of China, 2: 551-555.

Weimin Dong and Haresh C. Shah, Vertex Method For Computing Functions Of Fuzzy Variables, Fuzzy Sets and Systems 24 (1987) 65-78.

Y. S. Yun, S. U. Ryu, and J. W. Park, The generalized triangular fuzzy sets, Journal of The Chungcheong Mathematical Society. 22 (2009), no. 2, 161-170.

Ashok Kumar Shaw and Tapan Kumar Roy, Generalized Trapezoidal Fuzzy Number with its Arithmetic Operations and its Application in Fuzzy System Reliability Analysis, Int. J. Pure Appl. Sci. Technol., 5(2) (2011), pp. 60-76