PROBLEMS OF TEACHING METHODS OF THE SET CONCEPT IN MATHEMATICS

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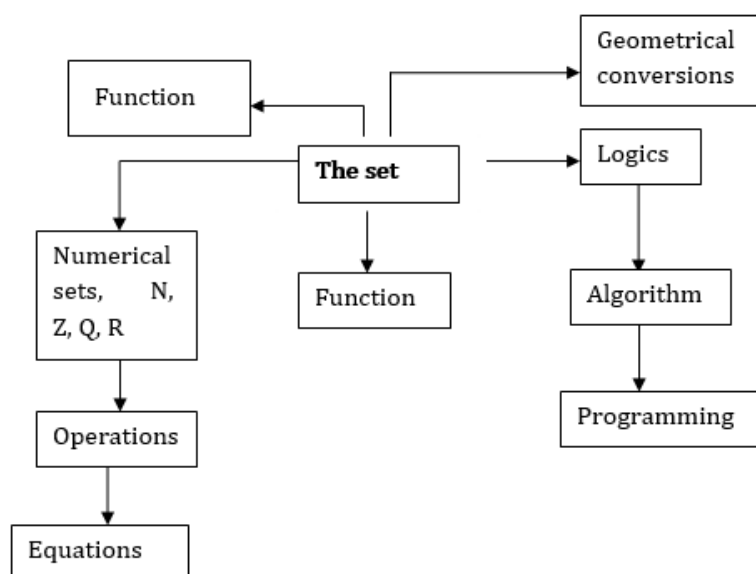[2] Faculty of Natural Sciences, University of Shkodra, Albania, Email: siditaduraj@yahoo.com

Abstract

Theme addresses the author's reflections and experiences during her work as a lecturer of mathematics (in general in mathematics and particular in calculus in several decades in Albanian universities, but also as formative lecturer of future teachers of this discipline of in cycles of pre university education, but also referring experience of colleagues from university region and the respective contemporary literature. The concept of set, as the basic concept of today math's school given intuitively from in primary school textbooks, and he treated in spiral form ,in our school, being back on every cycle until university. Scientific embodiment of this concept along with the scientific side also requires pedagogical skill that is realized by training and experience of this formative concept and teaching methods. Treatment is important, because sometimes not properly understood by pupils, but also by students who prepare for teaching. This brings consequences on other concepts that accompany mathematics program. It is delicate, as is always have the risk of falling into error. In this paper, are examined the mathematics textbooks of various cycles of Albanian schools (undergraduate and graduate) which treats the concept of sets and their operations, exploring different issues and ways to overcome them. Conclusions and recommendations will give as a teaching aid for teachers and pupils in the learning process of this concept, but even further, making valid topic for a broad measure of researchers at different levels of education

Keywords: mathematic, method, pre university education, set

Introductıon

The theory of sets, was born at mid-1800, it opened a new perspective for development of not only mathematics but also of other sciences. Since 40 years and more it has started to be taught at school by being an integral part of the curricula of all cycles of study. Before some year, the theory of sets was included by some special topics at the program of mathematics of all classes by starting since the first grade. Further, with the reformation of the curriculum of the secondary school, in general the specific topics for the treatment of the theory of sets, were reduced, but although its concept is a concept that it lies at the basis of a lot of other concepts in mathematics (the below scheme) that pose the essential mathematical formation of every pupil, its understanding remains at a great importance.

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The concept of the set is an initial concept in mathematics and it cannot be defined, but it can be described by relying to the concrete image that centenaries have rooted at the human conscience even though in vague contours and also in the rules put for the avoidance of the paradox that were flowing from the unclearness of the conception of the set.

We cannot think that in all levels of the school, the theory of sets is treated in its own right way from the axiomatic side. In general, at the curricula of schools it can be selected the way of intuitive treatment of the set, a thing that usually for the lack of attention of teachers that treat it or the way that is explained at the mathematics texts, it creates a vague concept at the pupils.

So, although at the mathematics program at the Secondary School, the hours on sets are being reduces, its concepts remains even silent, in basis of a lot of other concepts and also in the reasoning way of the students.

In this article we will analyze the way of treatment of the concept of the set and the operations on it at the secondary school by trying to give some recommendations in the way of their treatment with the purpose of the right concept creation at the pupils in relation with the sets and the operations on them.
What we will see are:

The meaning of the set concept, the statement of sets, recognition and right usage of symbolic "respect" and "included", defining the universal set and empty set, operations with sets, description of problematic situations.

The meaning of the set concept

In relation with the concept of the set G. Kantor (1845-1918) gives this definition:
"One set E is one defined collection of distinct objects that in our intuition we this as "whole". These objects he names as elements of set.

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In the definition of Kantor the word "in our intuition" leaves enough freedom as in nature of objects that can be thought as a whole also in the way that we can define a set, by listing its elements or by giving a characteristically feature that they can have and only they that we will name like elements of this set.

The word "distinct" in the statement of Kantor, implicates that after it is decided an equality rule according to whom to know that when these two elements are equal, in a set we might have also elements that are not equal, so they are distinctible with each other, for example $1 \neq 3$, but both are elements of the set of natural numbers. The word "defined" in the statement of Kantor, implicates that the set is one abstraction such that is entirely defined the way according to which every object we can say if it is element of it or not.

The pupils encounter with the meaning of the set since the first grade of elementary school. At the math's programs of the first grades we find as object the discovery of the mutual quality of one objects groups, classification on a group of objects according to the mutual quality (size, shape, color), the formation of the sets and their comparison.

So, the preparatory work, for the meaning of the sets has as objectives: observation and description of the objects, the discovery of the similarities and changes between the objects, their classification, objectives of the mathematics of the elementary school. Actually, the age 6 years old that the child enters into the school, is the age when he/she manifests the abilities of objects' analyses that enrich and mature with the development of his/her age. The main activities of the analyses are observation, description and comparison.

The observation and the description of the objects, discovery of the similarities and changes between the objects.

At the textbooks of the mathematics of the first grades at the elementary school, we encounter with topics like "Comparison", "order", "Recognition with the geometrical figures", "Something similar", topics that require the stimulation of observational abilities of children. The teacher should give to these hours a great importance. He/ she should request from the pupils to bring some different objects from home, where in class, in the form of a game, might askt their description according to the different physical qualities, functional or the position of their location. In the selection of the objects or pictures is important to begin with object very different (fig.1) or very similar (fig.2) and after to pass to objects where the similarities or changes are a little distinctive.
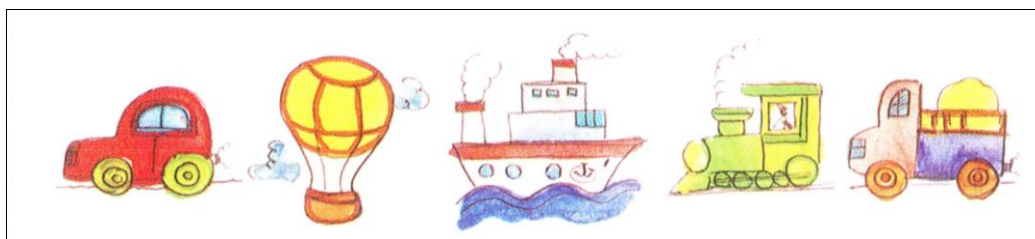

| Picture Nr. 1 | Picture Nr. 2 | Picture Nr. 3 |

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It is very clear that the finding of the changes stimulates the pupils to increase the capacity of analysis and the finding of similarities increases their general abilities, basic abilities for the creation of the concept of set and other concepts in mathematics and not only such.

It is important also the vice-versa process, by saying that the qualities of the objects, the pupils can find the requested object. This phase is important because pupils in addition to the observation of the objects make also the comparison of the qualities of each of it with the requested qualities by pushing the pupils to discover the mutual qualities between the objects which are distinctive between them.

If we notice the texts of mathematics of the elementary school, we can notice that the observation of the objects and the finding of the similar qualities of them, is passed so fast. In general, the presented exercises, there is requested to circle the objects with the given quality, without preparing the children that firstly they should know to analyze and to describe one object and later to discover their similar quality.

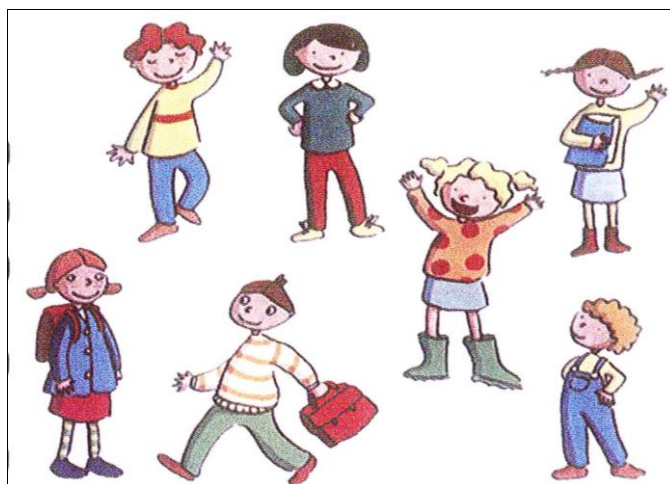*Circle vehicles of transportation with four weels*

Picture Nr. 4

Also, very important in the process of observation and analysis of the object is the formulation of the qualities from the pupils, whom are stimulated towards the enrichment of their vocabulary and towards to a more clear language. During the expression we might have complications of the situation because the children might classification a lot of qualities that might create confusion, they might not be clearly expressed by creation vague situations, or might use subjective features like "Beautiful", 'Small", "Handsome", "Good" ect., as in the picture:

Recalling the sets, indicate in the following picture if you can form set

Put **X** on **YES** or **NO**

	YES	NO
Pupils with red bag	YES	NO
Good pupils	YES	NO
Charming schoolgirl		
Students wearing trousers	YES	NO
Longest pupils	YES	NO

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Picture Nr. 5

In these cases it is important that from the teacher they should be oriented toward the selection of one criteria where everyone can rely on.

As a result, it is worth mentioning that the time dedicated to this action is an investment to the future when pupils will be dealing with more abstract objects like figures or geometric bodies, numbers and other features of them that will be given through the definitions.

The classification of objects

The classification of objects is an intermediate activity toward the formation of sets. In appearance is easy enough but pupils should make different operations to achieve the desired result. Currently we do not have only the observation of one object or the comparison between two of them, but the observation and the analysis of a group of them. The children by themselves should select the criteria on basis of whom they should list or classify the objects. The selection of the criteria presupposes the ability to compare the objects, to find the mutual similarities and changes between them. In this phase it is necessary include of labels which express the selected criteria or the characteristic features. This act begins since the first grade with the exercises of the type "Draw according to the label", "Relate the label with the right set" or "Complete the label".

The label is an essential step already for the reasoning of the pupils. It is not related with a concrete object but with something mutual with objects that need to be circled. This aspect becomes very apparent when we want to include the meaning of the natural number and the usage of different sets equivalent where the only mutual feature is the quantity of elements and in each of them we put as label the natural number that it belongs to.

The structure (construction) of the sets.

The structure of the sets passes through these steps:

1) The assessment of universe (The universal set).

It is important to define, list the object with which we are going to work or with which we are interested in, which will compose the universe by avoiding every possibility for confusion.

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At the recommended exercises should be presented clearly universe where it will be worked. The drawings should present uncertainties, is very important that the children should be certain for what it is about. At the first years of the elementary school we should not have universes with huge number of objects in that way that the students should control the situation and not to be distracted. It is good to change time after time the universe with the aim that pupils should understand that by changing the context, it changes also the possibility of classifying the objects.

In higher year when pupils are recognized with the numerical sets, the assessment of the universe remains important in all applications of the concept of sets. Since pupils start to get to know with the equation, in-equation is very important to emphasize the environment, universe where it is given and requested to be solved. For example in 4rth grade we find exercises of type "from set {2, 1, 5} find the root of equation x+20=22, or solve the in-equation 9x>45 at {3, 5, 6, 7}.

Normally the age of the pupils request that universe should be limited but from the teacher should be emphasized the fact that we request the resolve of the equation or in- equation in an environment, the assigned universe and are forming the sets of solution that might change with the change of environment. This is more highlighted in higher grades where the pupils learn the techniques of the solution of equations and in-equations. Preliminarily they should get to know with universe, environment where they will work. It would be recommended that the exercises should be formulated in this form:

Solve the equation 3x-2=0 in
a) Set of natural numbers N
b) In set of real numbers R

During the solution it is noticed that in set of natural numbers this equation has not root, so the set of its roots in N is empty, while in R this equation has one root $\frac{2}{3}$ and the set of its roots {$\frac{2}{3}$} in R has 1 element.

Actually not in all alternative books we find exercises that take the attention of the students in this direction. The authors and consequently the teachers stop a little when they have to solve equations and in-equations that contain variable in denominator. In this case it is being requested from the students to find the values that are not allowed and after the solution to check if the found values is part of the environment where we are solving the equation or not. The attention in this aspect is smaller and in general the pupils very fast forget to compare the found value with the values that are not allowed or forget to find the values that are not allowed.

Also in this way, in problems from life, where the variable x presents amount from the real world and as such it has limits, during the solution the pupils do not give attention to the environment where they work by giving in this way wrong solutions.

2) The definition of characteristic feature for the set we want to form which is a precise rule to identify the objects of universe that we want to collect in a set. So, we can change the position of the objects, the seclusion way of the objects with the same feature (surrounding lines, box, and letter) with aim that the attention of the children to be concentrated on the characteristic features and objects that fulfill it and not in the way of their representation. In elementary school at the children is created the idea that set is one amount of surrounding

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objects, because they in book encounter only with requests like "circle the set of…". In higher grades the teacher and text should use different way of giving the sets, as of listing the elements, representation of its elements inside the brackets {...} or A={x: x has the feature...}

3) To collect all and only those objects of universe that fulfill the rule, define of characteristic feature. All these objects are circled and are left outside the objects which do not have the defined feature. The taken examples from one side should inculcate well the process of formation of sets and from other sides should direct the pupils toward the special cases, where set matches with universe, where the set has only one element or does not have element, empty set. During this process through oral interpretations of the teacher and the dialogue with the pupils should be created the impression at pupils that the set is not the same with the meaning of the word group, bundle, collection, ect.

After is tis being work with the children with concrete objects can be passed gradually from one class to other class in presentation of elements of set through symbols.

All the process described above is in function of the creation of ability at pupils to distinguish if one element *belongs* to one set or another.

For this can be done exercises of two types. In the first case the teacher has constructed one set over a characteristic feature declared clearly enough for the student and has included in it one or more elements that do not have the declared feature, request them to find the foreing elemtns in the given set. In the second case, vice-versa presents a set that does not contain all elements and requests from students to make the correction by finding and put in set and those elements that should be from given universe, for example in the below figure
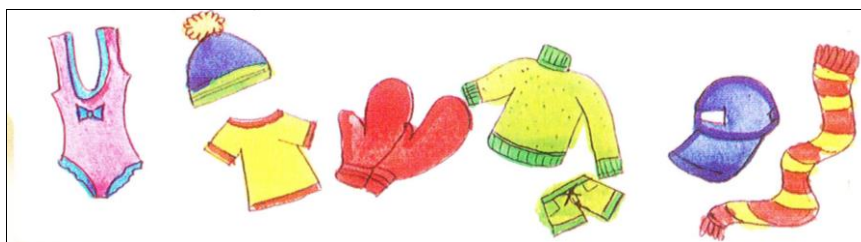
Only one of the two elements can enter in the set. Connect with arrow
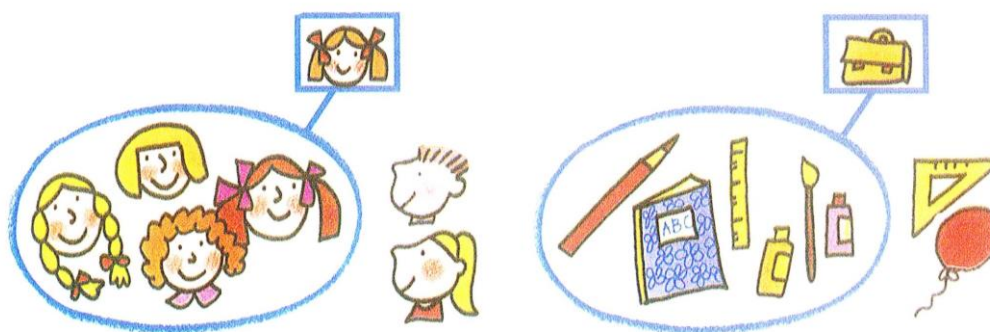
Picture Nr. 6

This activity is very important, because the affiliation of an element in a set is a well-defined relation through universal elements and the constructed elements. In very texts are usually put exercises of types "in below figure circle the set of…" whereas universe is taken the set of objects in the figure and as characteristics is taken the feature given in exercise. I could happen that the characteristic feature to no clearly defined as for example in below feature:

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Circle clothes that we use in winter

Picture Nr. 7

Which can limit the pupils, because some can circle for example the hat, some not.

Actually, when we get to know the universe and the characteristic features is simpler for the pupils to make the set than the vice-versa activity; we know the universe and set and they find the characteristic feature of it. During these exercises can be given even some cases where the solution is not only one.
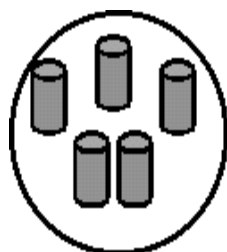
Picture Nr. 8

During these exercises the students reason with so called "reasoning from the opposite". During this activity pupils should understand the right meaning of specific temrs: *universe, set, elements, all and only, belongs, does not belong.* Is the teacher whom should use these expressions in the correct way and request occasionally to the pupils to use the language of sets in change from everyday language.

All this is not a simple process. Only the term "element" requests time to be included in everyday language because it presents the process of disconnection from the concrete object. It is a way of generalization. Also should be careful from the usage of some quantitative adverbs: all, some, every, none ect.

The core of meaning of set according to Kantor and visualization of collection of objects as a whole one.

Actually for the pupils, the difficulty remains in the consideration of the collection as the whole one because for them is a new meaning. They pass easier the difficulty of creation of se, after some time they intuitively understand set of different objects, understand their elements, but have difficulty enough to understand the set as one element. However intuitively during their development they encounter with concepts where it is necessary that the whole should be thought as one. They think for their class as one only, or they think for their football team as one only despite the fact that the players on it might be different, when the team goes in field, it is a set of 11 elements that can be thought as one in whole. By

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learning math's, since early they understand the line as one in whole but alter they learn that, that set of solution of equation ax+by=c, where a, b, c are real numbers and x, y are variable, also might find some elements of this set. Also when they learn the concept of functioning they understand that the function is a relation between two sets A,B that fulfill the condition that every face cannot have no more than one reflection. Usually they do not keep in mind this definition, we do not know to have a function f: A$\rightarrow$B and we have triple (A, B, G_f), but they concept it as one thing and do not get surprised if we talk about the features of function f , when we say that the function f is continued, by considering it as one thing.

Also, should be careful because not only one as whole should be thought as one only is part of the set of its parts. Is it a set a table watch? If we analyze it in composing pars and put them in a box, and name "the set of watch parts" is clear that it is not a watch anymore. So, the watch is not a set of its compounds. Also the triangle is not a set of 3 segments (that can be its ribs).

Difficulties that pupils have on understanding the set as whole, goes out when we request the set to have as elements set, set f subsets of one set, set of equivalent's classes of the relation of other equivalent. This difficulty is increasing when we deal with infinite set.

The representation of se with Venn's' diagrams

When a pupil takes out a line to circle one collection of objects, it makes an important and significant operation that takes him/her toward the representation of sets through a plans figure closed that presupposes that inside it are elements of set.

A such presentation of set was being represented for the first time from L. Euler (1707-1783 and later from J. Venn (1834-1923). Diagrams of Euler-Venn have been stimulations for mathematicians especially in studies in geometry, combinatory or theory of graphs.

Whenever we give the meaning of operations with sets, the joining of set, cut, fulfillment, different ect, we illustrate them throughout the diagrams of Venn. Also when we want to give meaning to the operations with numerical set in R in all texts is used their schematic presentations throughout Venn's diagrams.

The presentation of sets with Venn's diagram helps the pupils in perception of different sets. However the Venn's diagram should not be confused with the concept of set. It is a visual appearance and not only with a lot of limits. It is important in understanding the concept of set, it is the meaning of characteristic feature which can be expressed with words or in symbolic way. There are many cases where the appearance of set in Venn's diagram gives confusions to the pupil. For example there are some set that are being cut. The drawing with the Venn's diagram for each of them to have a perception for their cut brings confusions more than helps the pupil to percept the thing that is being requested. Or if we want to represent though Venn's diagrams, set, elements of which are set. Even here we can have confusion because pupils are taught that set can be showed as a zone in a plan surrounded by elliptical line and its elements are as points in this zone and now how should they represent the set hat are elements of the given set?

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Conclusions

As noted above the concept of the set is an initial concept and on it lies the nowadays mathematics. But the acquisition of the theory of set since the secondary school is difficult of the pupils, so we cannot request its full representation in the programs of math's of secondary school. However the exercise of the activities as described above take us to the formation of the concept of the se to be done from the teacher and to stimulate frame the texts of mathematics. They are teaching activities that stimulate the healthy reasoning at pupil and make him/her able of acquisition of knowledge by passing from collection notion, of set, of number and all notions that are being taught at school.

References

K. Kuratowski, A. Mostowski. *"Set theory"*, 1970

Elida Hoxha. *"Bashkësitë",* 2007

R. Ferro. "*La teoria degli insiemi", p II, L'insegnamento dello matematica e delle sienze integrate,* 16, 11-12, 1077-1099.

Giorgio T. Bagni " *La rappresentazione degli insiemi: appartenenza e inclusione" Atti del Seminario Franco Italiano di Didattica dell' Algebra, VI*, Universita di Torino, 2008.

E. Xhaja dhe F. Kodheli, *Matematika 1*,.

Kristaq Billo. *Matematika 3*,

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