# The complex network methods used in the study of some data series for real systems.

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**Abstract**

Real system's behavior studied with the complex network methodology sometimes can result on remarkable shortening of the research path. We studied some typical complicated systems as opinion formation networks, pollution particles and irregular activities on voting systems considering their distributions against some parameters of interests and from standard theory we proposed possible mechanisms shedding light on commonly questionable sources of typical behavior. The confirmed results are used to simulate some other quantitatively unknown or hidden phenomena as administrative corruption. Using complex system methodology we seeks for alternative study of complex phenomena as on aerosol and pollution particles distributions.

*Keywords: complex systems, self organization, voting irregularities, corruption modeling, pollution, distribution.*

## Introduction

Statistical study for some real multi particle system has been considered straightforwardly as framework of complex network approach. The network is a graph where the nodes are particles and links are interactions. Therefore if a graph is subject of math's, the network is more physical object. The most important thing on the network is the existence of interaction. Usually the interactions are converted on mechanisms. The dynamic of such systems cannot be always reduced to mathematical equations, even from lack of pure deterministic behavior, even from the high fluctuation rate around the average values. The statistical point of view considers the distribution to see the interior of the system and mechanisms. The topology does matter too but sometimes it can be reduced on the modification of mechanisms. The fascinating behavior of particular interactions and the geometry of the graph will lead to particular well known distributions: co-authorship and artist networks show a power law distribution the same as the distributions of large cities and internet connection to specific pages, so does the avalanches and earthquake amplitudes densities, distribution of defects and rupture will show a Waybill shape and so on. There is no apparent natural similarity of such systems and behavior but the distributions show that they represent similar network.

## Distributions in complex systems

Thermodynamics is based on an equilibrium approach which is a very particular state of systems among common real ones. Some linearity hypothesis routinely is proposed and physics tends to avoid non relevant dynamics for better understanding the most characteristic behavior. Under such reduction systems behave well and do show us the most probable state characterized by intensive parameters-that is thermodynamic approach. Complex systems are much more complicated but even here physicist does make good progress, we're dealing with distributions for some meta equilibrium state and even no equilibrium at all. The basic idea is to use the distributions as an intermediary step in understanding dominant mechanisms that are very complicated to be known. If we consider an agent based model for a given system of particles, the probability for link establishment P(i,j) if not random (no cause) is defined from the mechanisms of node interactions. The cumulative result could give rise to a stable distribution of the property "the node i has k links" if some self organization. Here the so called "voters model" is very fruit full to understand the behavior and dynamics of the network. The voter model is a network with opinion particles of the nodes of the graph, characterized by two states "agree=1" "disagree=0" and the links $i_{\rightarrow}j$. There is a rule on the system: if a link is established, the opinion of nodes j (waiting) is the same as the opinion of the incoming node (i) ore vice versa. The opinion on the system looks very like magnetism and this why physicists continue to be on the top list of successful studies for such system. Diverse spin system application here has shown. Therefore if the interaction between particles is governed by the rule "the higher the number of links k for a node j is, the higher for a new link to incoming node is" the distributions result a power law commonly found on human activity networks. If there is particle that never change their opinion state, one leads to a Gaussian distribution, if there are heterogeneities all other constraints on the system the distributions is exponential, and if drifts, friction or forces are present on the system the distribution will be Levi or more complicated. Other phenomena will push the system out of the stable distribution.

Our recent work deals with electoral opinion formation in Albania. Here we found that the opinion formation is better described by a two graph network each characterized by its own mechanism of links establishment. In the first graph we found that voter's decision will be governed by a preferential attachment rule that produce a power law distribution. Roughly speaking on this region we have up 20% of votes that is clustered here are limited up to this boundary. Here one has a specific network, where the graph is directed and the attractive centers (subjects or their candidates) are seen ranked. The next graph differs only by mechanisms and topology but we assume this last on modified preferential mechanisms with zealot effect and median field present on the system. The distribution in this region is found Gaussian with a log-normal part of the very high fraction (>85%). Simulation based on those empiric assumptions does support those mechanisms (fig.1). We emphasize that there are some evidences on mechanisms, therefore the mechanisms selected is proposed straightforward.

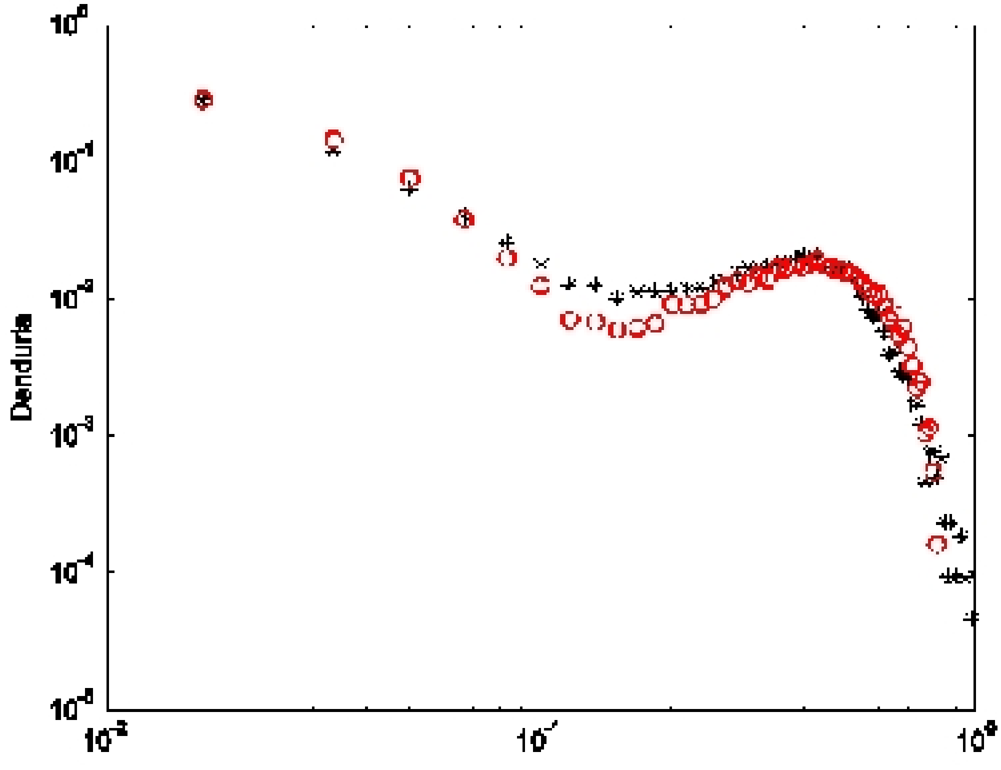


Fig1. Distribution of densities of links on our electoral network. Results of voting by VCC, parliamentary election 2009. The red circles, real pints, black asterisk, simulated ones.

We focus our research on the networks and systems where no clear evidence is found therefore assumption and hypothesis can be questionable. The challenge will be highly scored as there are no other apparent means to identify particular or dominant mechanisms among potentially many of them.

**Identifying possible mechanisms on some exceptional voting subsystem irregularities.**

It is very intriguing going deeper on a socio-political very questionable issue in our election, the mechanisms that govern irregularities. The administrative procedure failed recently to find the correct answer as where has no scientific proves provided to support the arguments leaving the discussion on the political arena. Here we offer another view. Firstly we consider the natural irregularities as invalid ballots. If the process will happen naturally that is with no apparent cause we expect that the number of invalid ballots should be normally distributed around a median common error value (somewhat 1-2%).

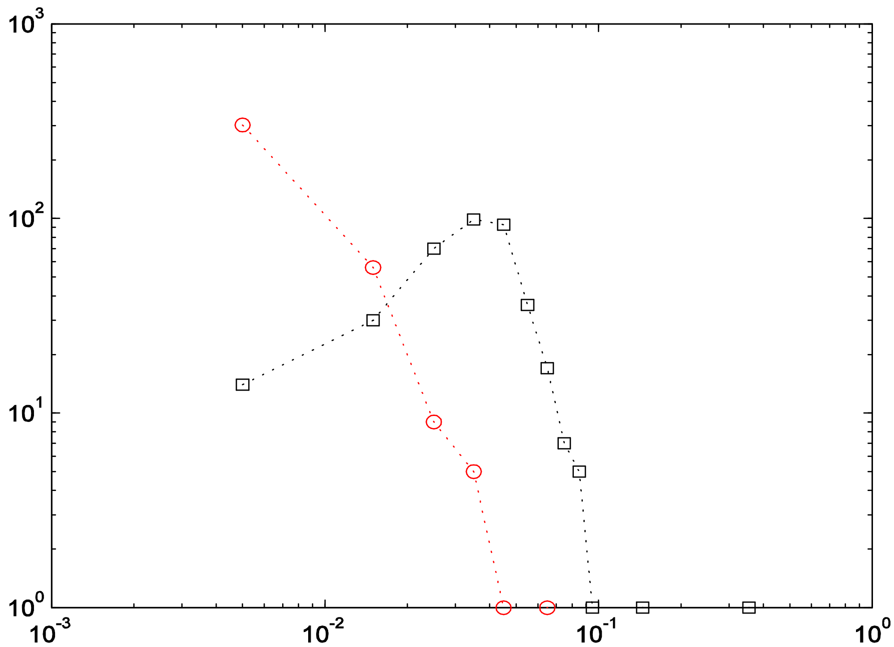


Fig2. Invalid ballots (circles) and irregular ballots (squares) (2003).

The Gaussian like behavior of invalid ballots tells that no mechanisms is involved, the common error of voters leads to invalidity of ballots (invalid is a ballot that has no velar voting sign etc.). The irregular ballots are distributed as power law making evidence of any self organized behavior. It is clear that the errors of individuals can not be correlated; therefore the correlation is imposed on the system. We assume that this can be related with "Wrong Box Voting" which itself can be affected by permanent factors. The *Wrong Box Voting* irregularity is funded on parallel voting as on 2003, 2007 and 2011 for local election and 2005 for parliamentary voting. There is no record on official books or files for those irregularities, so we use some indirect estimation for the phenomena. We calculate the differences on valid and invalid votes for majors and counsellors. The sum of valid votes and invalid votes for two types of voting should be the same, so differences must be zero. If there are differences, it comes from wrong ballot voting. Our study therefore the differences $\Delta_{k-s} = \left(V_{vl} + V_{pvl}\right)_k - \left(V_{vl} + V_{pvl}\right)_s = \Delta_{k-s}^{vl} + \Delta_{k-s}^{pavl}$ that could count for wrong ballot voting. Making this cross evaluation we see that differences on total votes is very likely power law form and all other differences show a power law part. Therefore the irregular votes mentioned above will be defined as the evidence of wrong ballot voting, and is characterized by self organization behavior. The explanation on such correlation (self organization is due from long range correlation) should be searched on common cause for such event to happen. First the discrimination of ballots could have been difficult as from the miss collared labels for ballot boxes, or misplacement of them. Second possible voluntary miss orientation of the appointing person on the VCC could have been the place too.

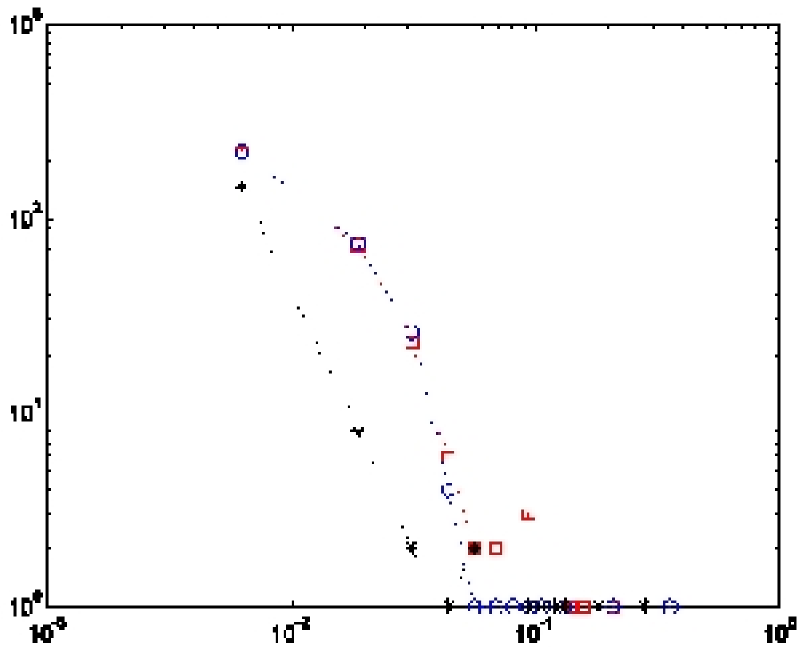

Fig3. Distribution of deformations: total differences of invalid ballots by circles, differences of valid ballots by squares and the total differences of votes found on two types of voting by asterisk.

A very interesting irregularity perhaps specific on our election is the wrong tabulation phenomena. The votes for an election zone (ZEC) are summed up for the results of voting unities (VCC) that is the difference should be zero, but it is not zero in 2005 and 2007. The idea is that during the process of tabulating the commissioners unlawfully displace the votes from subjects X (Y to reach the threshold for a mandate. In 2005 this displacement is due to the fact that on a coalition the principal subject doesn't profit from proportional votes and voters did not realize to synchronize with tactical voting appeals. So "clever" militants in commissions decide to complete the mission themselves. This is clearly an organized behavior and the distribution is a power law (fig. 4). It is quite clear that this evidence is from the nature of the distributions and stands physically pure in this form irrelevant of documentation techniques or other reason that having random and occasional occurrence can not produce a power law distribution.

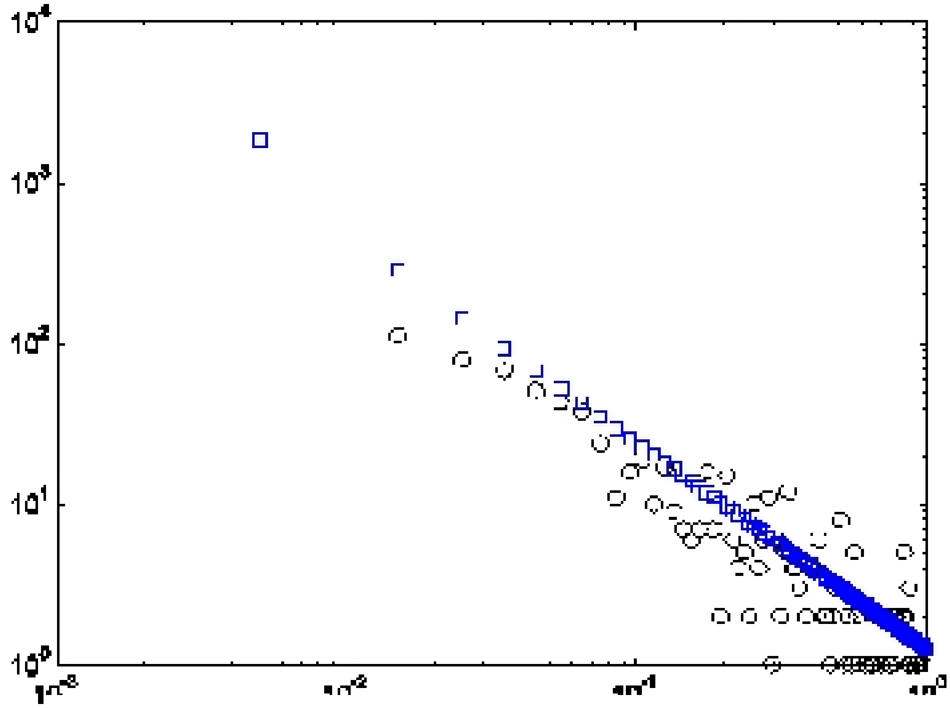


Fig4. Distribution of tabulation errors for the proportional part of the results on the parliamentary election 2005

**Some other application: a possible social problem simulation-case of corruption.**

Political opinion formation studied can be considered indirectly as a tool of characteristic topology and interaction on the opinion formation process. Hence we can construct a network to describe the spread of the corruption or other processes on the society attempting to catch some quantitative elements. We propose that the structure of the society network and constitutive groups could be he same as the one identified by a fitting modelling process of the electoral opinion formation. The decision of citizens and bureaucrats to be involved in corrupted activity will be governed by some utility optimization rule. We choose the network with some hundred citizens asking for some issue to the public offices and about ten times less officials responding to them. First we have to make a link between elements of two groups. Each couple will develop the analyses based on the results of the payoff matrix with elements that take the maximum on combination {corrupted bureaucrat, corrupted citizen}. Therefore we make a preliminary selection of two groups to count for the real refuse of some peoples and officials to participate passively or actively in illegal behaviour. We set the honesty index for official around 0.4 from corruption index perception (3.8) divide by 10 as the opinion's value are selected by pseudo-random numbers (0-1). The honesty index of common people is taken between [0.6,1] assuming that common people are less corrupted than officials. Now we run the activity. Individuals come to the service office and some of them seek for someone to offer bribe to complete "the shortened service", so we deal with ones that are wishing to be corruption-active. We start with random (uniform) distribution controlled by barriers. We don't consider the tunnelling phenomena, imposing the constraint "if corruptive parameter C is smaller than a barrier, the activity is impossible (no link is established). For all offices we admit the corruption is possible but not all servants are corrupted. Therefore the number of corrupt officials is diminished by imposing barriers which can be from zero (all corrupted) to one (no one corrupted). Here another phenomenon must be considered: there are beliefs that work like fields on a magnetic system. The effect of common belief is considered as additive or multiplicative term on the probability of link establishment or on lowering the social barriers. We import the specific of our society network from political opinion-formation appointing the exponent of 0.705 that is the most common exponent found on the majority voting in Albania. If the exponent will be selected for 1, the distribution would be more flat, that is not the case reported from our previous evidences. The process of node selecting is organized by a random choice.

This is most supported idea for low level administrative corruption, that is bribing directly and not involved in organized corruption.

Next the number of initial collaborators is introduced. We set three groups of corrupted officials, the having 3, 2 or 1 individuals helping in finding bribe offers. We see that if no collaborator is involved and no field is present on the system, the number of officials that will profit from corrupted activities shows disturbed power law behavior. The simulation is performed using the modified preferential attachment mechanisms with exponent as found in a political opinion formation (Alfa =0. 805) and by assigning the corrupted officials with collaborators very likely militants on our previous works.

**Opinion dynamic approach. The role of common negative beliefs or fields.**

Next we used Wedilich approximation on opinion dynamics to calculate the dynamic for high stage of corruption that is the case of corrupted bureaucrats that have concluded with many links on first simulation. Here the rules are different. After having released some

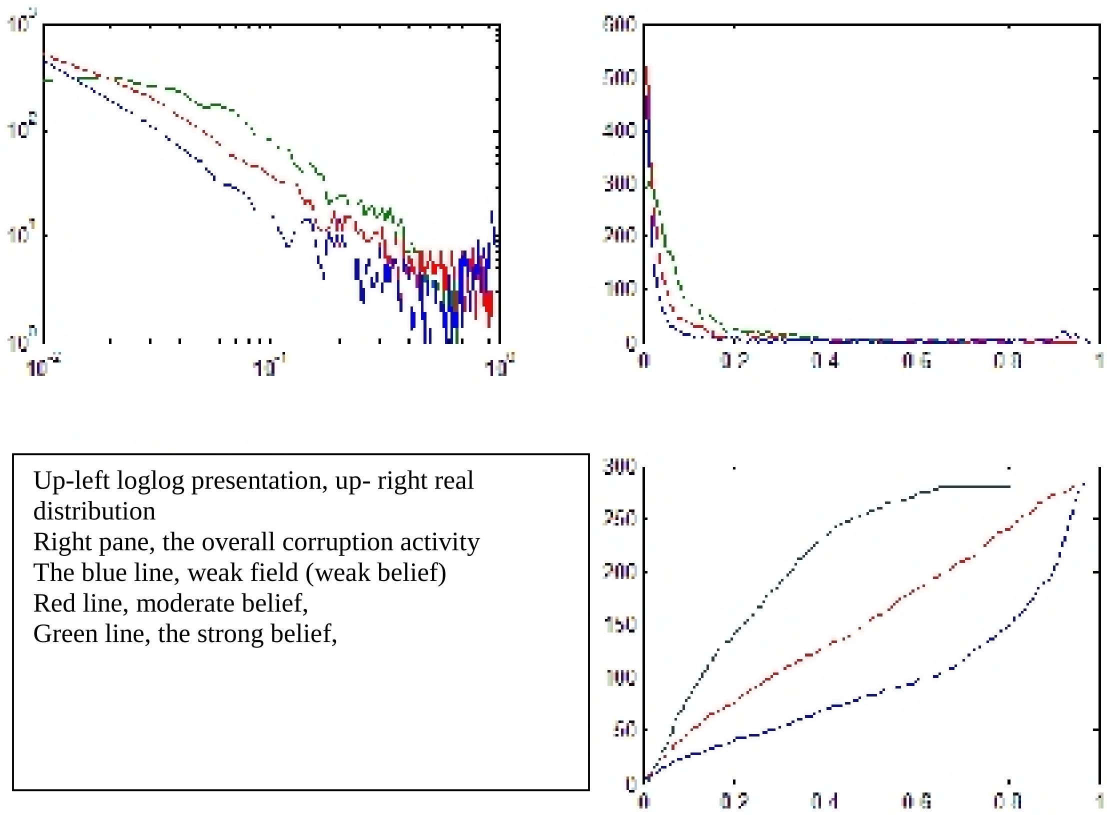


Fig 5. Simulation for corruption activities.

good benefit from banal corruptions, peoples will follow another route on their own. Some clever will quit or continue to profit from the low level corruption activity as described above. We assume that participants in the game interact, exchanging value at risk to their own profit. They will admit to going further (o=1) on the risky game, or refuse and standoff aside (o=), making us possible to model a small network by the binary opinion site. Weidlich realize to engineer an elegant model based on binary opinion formation, which we use to apply with some modification. Here N officials that were produced by the first mechanism will face the alternative on the next step, to go or to stop depending on the results of a dynamic analysis, unconsciously adapted as a natural reaction under risk events. One takes firstly a global variable as the normalized change between group1 and group2 that act together: $y = \frac{n_1 - n_2}{2N} \rightarrow -1 \leq y \leq 1$ where $n_1$ and $n_2$ stands for each group respectively and N is the number of traders. Second normalized variable represent the individual interior inclination. The personal preference variable to enter or not in the game will be appointed as the x variable of political preference on the Weidlich reference model. By straightforward and ingenious physical concept implementation one achieved to the equation of dynamics

$$\begin{aligned} \frac{dy}{dt} &= \sinh(\text{©}y + \text{¥}x) - y\cosh(\text{©}y + \text{¥}x) \\ \frac{dx}{dt} &= \text{«}\left[\sinh(\text{©}y) - x\cosh(\ \ y)\right] \end{aligned} \quad . \grave{} \qquad (*)$$

Whereof κ represent the “conformism”, γ the “dissident” weigh on the system and β the inclination for one’s of two opinion value toward the participation. It looks like a specific game on the specific trade. The stability analysis for equation (*) gave very important information for the system. If parameters on the model will be changed on specific value, the dynamic is very interesting giving rise to phase transition toward the corrupted system. We see that for specific parameters on the system my=2;g=1;k=3.5;b=-4 the situation will develop toward an idealistic state where all want to engage but no one can do it. Here we stray from what state (y=eps, x=eps). Next we change the parameters giving the value k=1.5;g=1;b=-4;my=2 . After some time the system will fall on the state where two groups equal and individual preference seems to be neutral. (fig.7, right left corner).

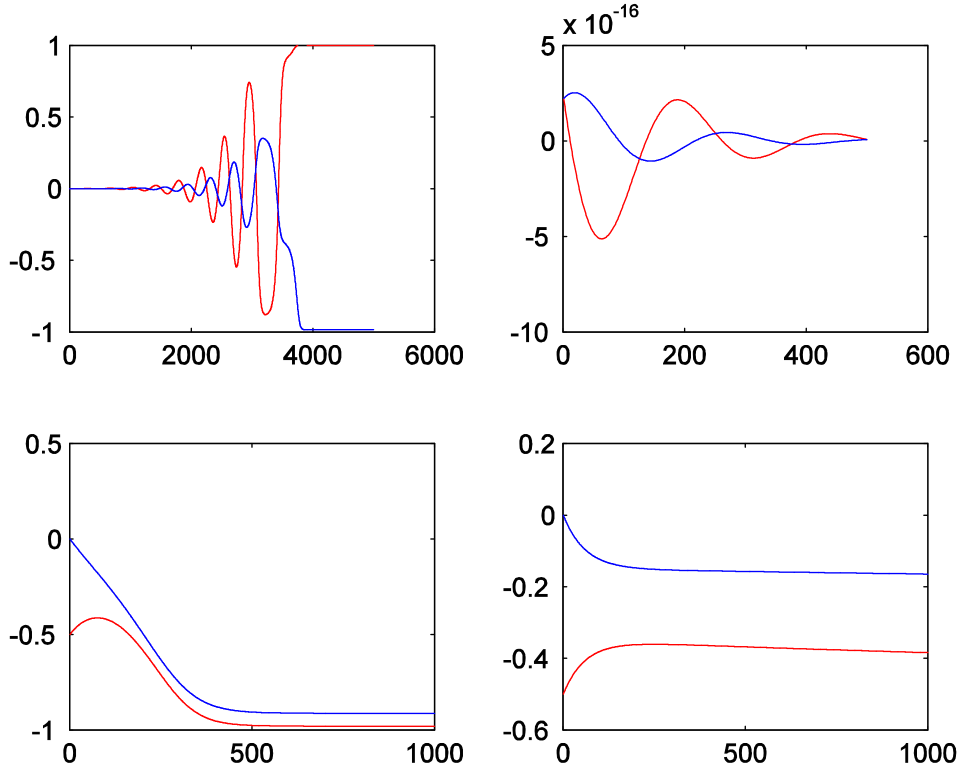

Fig6. Dynamic of individual preference (red line) and participation misbalance.

In this situation, the distribution found in the simulation above will keep its form unchangeable. If we take a specific choice of parameters at k=1.15; g=0.5;b=2.5; my=0.5; ny=0.5, the final state will converge toward saturation where both preference and the overall numerical inclination reach their extreme value (xf=-1,yf=-1. It is interesting that an intermediate value of parameter k will realize a partial saturation of two variables

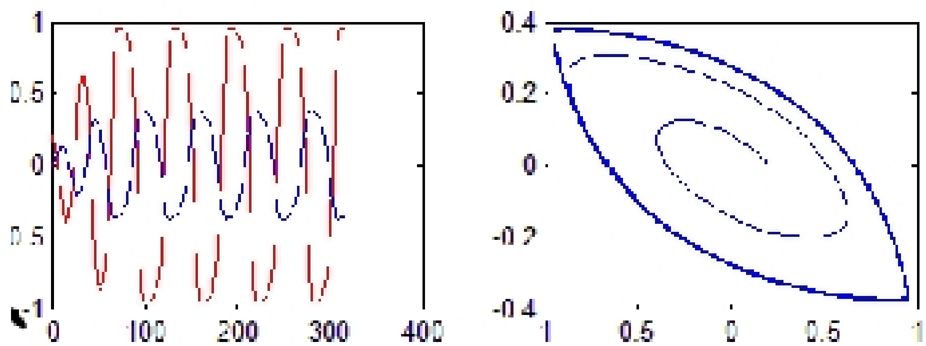

Fig7. The oscillation study state

Using the definition of parameters, we see that under common belief or idea (as prescribed by parameter b on the model) among officials the high value of differences for continuing on corruption or refusing it, higher value of characteristic engagement is found. Not surprisingly the effect of fields or common beliefs are enough strong to push the system toward strong oscillation or to bring it on steady state (0,0) point. Therefore reducing progressive corruption process one needs to remove the common belief from the system that can be realized by political and judicial mechanism out of the area of this research.

**Aerosol conglomeration and other pollution's particle distribution**

Atmospheric aerosols originate from the condensation of gases and from the action of the wind on the Earth's surface. Fine aerosol particles (less than 1 mm in radius) originate almost exclusively from condensation of precursor gases. A key precursor gas is sulfuric acid ($H_2SO_4$), which is produced in the atmosphere by oxidation of sulfur dioxide (SO2) emitted from fossil fuel combustion, volcanoes, and other sources. H2SO4 has a low vapor pressure over $H_2SO_4$-$H_2O$ solutions and condenses under all atmospheric conditions to form aqueous sulfate particles. The composition of these sulfate particles can then be modified by condensation of other gases with low vapor pressure including NH3, HNO3, and organic compounds. Organic carbon represents a major fraction of the fine aerosol and is contributed mainly by condensation of large hydrocarbons of biogenic and anthropogenic origin. Another important component of the fine aerosol is soot produced by condensation of gases during combustion. Soot as commonly defined includes both elemental carbon and black organic aggregates.

The mechanical action of the wind on the Earth's surface emits sea salt, soil dust, and vegetation debris into the atmosphere. These aerosols consist mainly of coarse particles 1-10 mm in radius. Particles finer than 1 mm are difficult to generate mechanically because they have large area-to-volume ratios and hence their surface tension per unit aerosol volume is high. Particles coarser than 10 mm are not easily lifted by the wind and have short atmospheric lifetimes because of their large sedimentation velocities.

Coarse particles emitted by wind action are similarly removed by rainout. In addition, they sediment at a significant rate, providing another pathway for removal. The sedimentation velocity of a 10 mm radius particle at sea level is 1.2 $cms^{-1}$, as compared to 0.014 $cm\ s^{-1}$ for a 0.1 mm particle. The bulk of the atmospheric aerosol mass is present in the lower troposphere, reflecting the short residence time of aerosols against deposition (~1-2 weeks). Aerosol concentrations in the upper troposphere are typically 1-2 orders of magnitude lower than in the lower troposphere. Based on direct microscopic observations of the morphology and composition of urban particles, Whytlaw-Gray and Patterson (1932) proposed that coagulation should be an important mechanism shaping the size distribution of atmospheric aerosols. Later, Junge (1952) also concluded that the lower end of the aerosol size distribution, below 1.0 μm, is determined by the balance between the sources and coagulation decay of the particles. Numerous studies since then confirm that coagulation among the haze particles, particularly below 0.5 μm, is an important process shaping the size distribution. The removal of large particles by sedimentation is a plausible mechanism that determines the shape of the size spectra for the range above a few microns. The atmospheric lifetime of coarse particles is also determined by sedimentation. The first physical explanation of Junge's power law size distribution laws was provided by Friedlander (1960). He proposed that the atmospheric aerosols exhibit self-similar and power law shapes because the input rate of fine particles from various sources is balanced by the course particle removal rate by sedimentation. He also proposed that coagulation provides the mechanism for the transfer of particles from fine to coarse sizes. Taking the analogy from the cascading transfer of turbulent energy from large to small atmospheric eddies, Friedlander termed the equilibrium size distribution, balanced by coagulation and sedimentation, as the “self preserving distribution.” Observing the rays of the sun soon after a rain, Rafinesque (1819) was forced to the conclusion, as many “philosophers” before him, that in the “chemical laboratory of the atmosphere” there must be “in situ” processes responsible for the haze visualizing the rays of sunlight. Tyndall (1869), Aitken (1888) and many others confirmed that chemical aerosol formation occurs in the atmosphere at significant rates. Haagen-Smit (1952) in his key contribution to the chemistry and physiology of the Los Angeles smog documented that man’s activities contribute significantly to this chemically formed haze. The roles and interaction of nucleation, coagulation, and condensation in the dynamics of the smog aerosol were established and numerically formed by Husar et al. (1972). Particles of sub-micrometer size collide with air molecules randomly and behave collectively as a gas. Altitude profile of aerosol concentration abbeys to Boltzman distribution $n(z) = n(0)e^{-\frac{mg}{kT}z}$. Random collisions on the micro scale give rise to diffusion on the macro scale. At the equilibrium state, diffusion and gravitational convection must balance each other so that $-D\frac{dn}{dz} - nV = 0$ where V = fall velocity. Equating Stokes drag with the particle weight $6\text{®«}aV = mg$, we get $V = \frac{mg}{6\text{®«}a}$ and upon comparison with previous equation, the Brownian diffusivity can be identified $n(z) = n(0)\exp\left(-\frac{mgz}{6\text{®«}aD}\right)$ where $D = \frac{kT}{6\text{®«}a}$. The diffusivity of aerosol particles is $D = \frac{0.325 \times 10^{-11}}{a}$. In the case of gas molecules the mean free path is $l \sim 10^{-5} - 10^{-6}\,cm$. When small particles bounce around randomly by surrounding fluid molecules, they may come so close to one another that Van der Waals force binds them together. This is coagulation. In moving fluid additional factors such as fluid shear and Coloumb forces may intervene. A simple model (by Smoluchowski) for a stationary fluid with identical spherical particles of radius *a* is as follows.

| $2a$ ($\mu$m) | D ($cm^2/s$ ($T = 20°$)) | V (cm/sec) ($\rho_s = 1g/cm^3$) |
|---|---|---|
| 0.001 | $5.14 \times 10^{-2}$ | |
| 0.01 | $5.25 \times 10^{-4}$ | |
| 0.1 | $6.75 \times 10^{-6}$ | $8.62 \times 10^{-5}$ |
| 1 | $2.77 \times 10^{-7}$ | $3.52 \times 10^{-3}$ |
| 10 | | $3.07 \times 10^{-3}$ |
| 100 | | 30.2 |

Table 1. Diffusivity and settling velocities of aerosol sizes

Let us focus attention on a fixed particle. Consider a spherical shell from r to r + dr.

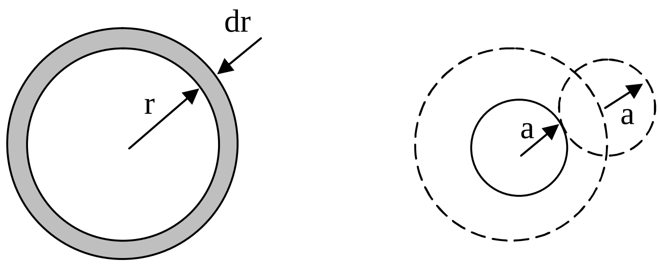


Fig. 12. Schematic presentation of coagulation process between aerosol particles

The rate of increase of particles inside the shell is $\frac{\partial n}{\partial t} 4®r^2 dr$ . This must be equal to the net influx through the two surfaces of the shell $-\frac{\partial n}{\partial r}\left(-4®r^2 D \frac{\partial n}{\partial r}\right)dr$ . Thus $\frac{\partial n}{\partial t} = \frac{D}{r^2}\frac{\partial n}{\partial r}\left(r^2 \frac{\partial n}{\partial r}\right)$. Putting the initial conditions, we obtain $n(t) = \frac{n(0)}{1+Kn(0)t}$ where $K = 16®aD = \frac{8kT}{3«}$ is the coagulation coefficient. The specific values of this coefficient are presented in table 2.

| a (cm) | $10^{-7}$ | $10^{-6}$ | $10^{-5}$ | $10^{-4}$ | $10^{-3}$ |
|---|---|---|---|---|---|
| $K_o \times 10^{10}$ $(cm^3/sec)$ | 323 | 34 | 5.56 | 3.19 | 2.98 |

Table 2. Coagulation coefficient for several aerosol sizes

As the nature of the phenomena herein is well known, we propose a complex system alternative to distinguish between competitive factors on particle density evolution. Our recent work on the evidences of pollution shed light on the situation of atmospheric and air contamination of Albania. Here the interest is to see what is underlying the evolution of particle distribution on the air, if the systems does show some complex behavior. Under general assumption we consider the distribution as an indicator of phenomenon that causes the change of the fraction of pollution or foreign particles in the air. We argue that as far as the distribution will approach to a specific function, the more possible is that a particular mechanism is involved in the process of the change of such fractions. Therefore the dominance of Gaussian distribution will identify the fact that sources of particles affect randomly the system. Otherwise if the distribution looks to be power law it is possible that some self organization process is present on the system. This will be the case of dust particles or some aerosols that can get together with a massive grain to reduce the number reducing the probability of finding big particles on the air. The graph on fig8 shows the distribution of number of particles reaching the monitor each minute on the same place and same level from ground. We take care on the binning process as the first step of distribution construction form the data series. So the selection of binning is computed according to the error limit that is we do not goes below one percent on the presentation even the graph do resist on a wide range, making practically this step free of subjectivities.

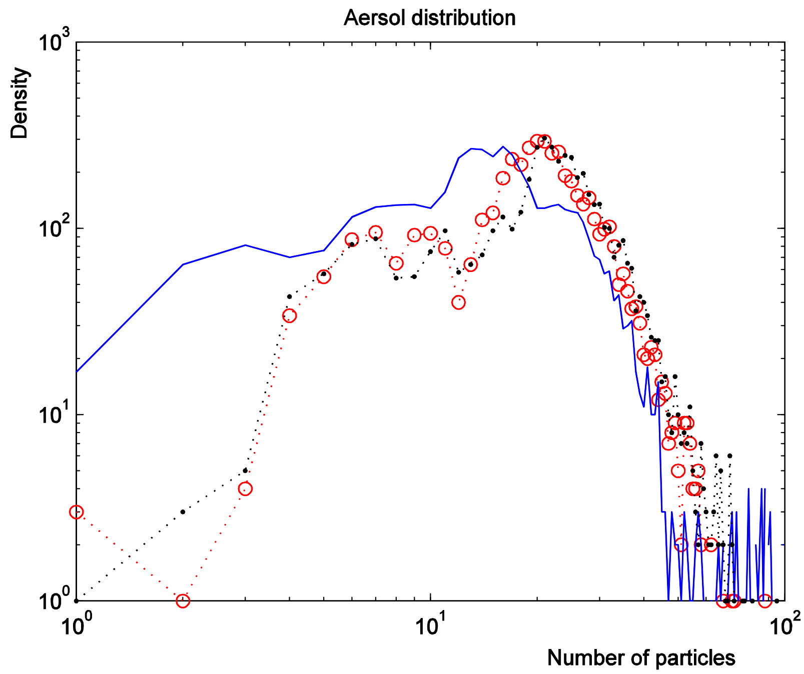


Fig8. Distribution for particles per minute registered Chanel 0.25-0.45 micrometers : red color, 0.25 micrometers, black color, 0.35; blue color, channel 6

Therefore the distributions are found specifically related to the channels that is the radius of aerosol or other pollution particles. Small particles do shows deformed Gaussian distribution which begin to evolve immediately from the channel 2 (r~250 nm). On the range radius 0.4μm the distribution is better fitted to a lognormal and this keep changing reaching a near to power law distribution for heavy particles. (Fig10). Generally speaking the presence of power law or deformed lognormal (that is with large variance) could indicate the self organization behavior interaction sources. The process is very complex as distribution transform continuously (within common sense as only few channels are available on registration process). It is not possible to speak analytically but we comment the result. So it is possible that a good approximation could be a more generalized distribution as from Levy processes where of Gaussians, exponentials or power law belong to a large family of distribution as parametric exponentials are. Therefore a stochastic process of particle generation is dominant for small radius pollution mass. This will be added to stochastic noise resulting to a near Gaussian distribution of the number of particles. The net result is the distribution where only sources are involved and they produce randomly. The coagulation process that is prescribed by a power law of time cause deviations as remarked on the graph on the Fig8. Periodically those production-conglomeration oscillation are overlapped giving rise to a stochastic regime as on the fig9. But for larger particles the process of coagulation seems to be accompanied by other interaction where a simple fusion is possible. The process can be intensive enough to overcome the other proportionally with their mass and radius. The net result will give a near power law distribution as found on channels above 30 (particles with radius greater than 0.032mm)

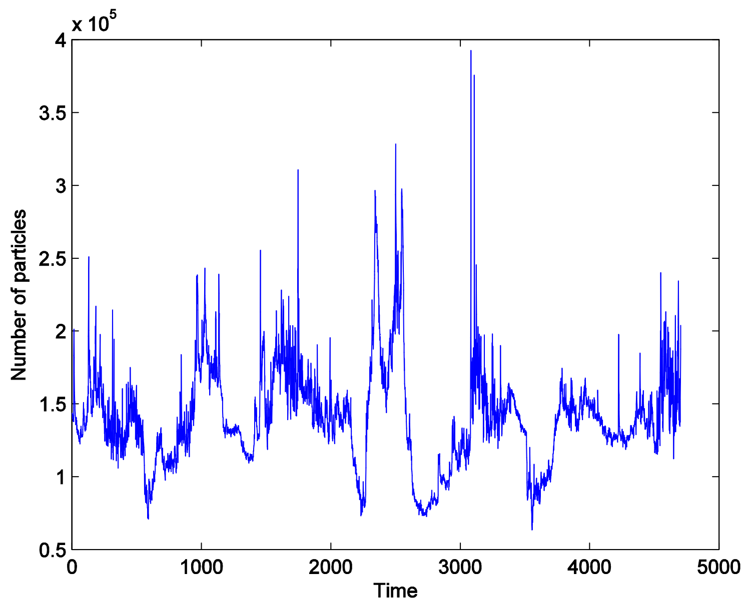


Fig.9 Near stochastic time-evolution of particles registered (chan1.)

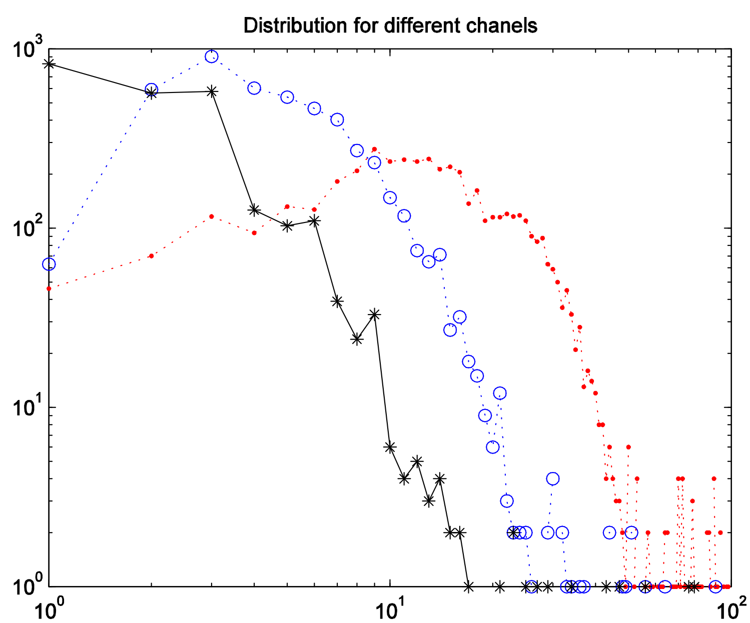


Fig.10 The evolution of distributions for different channels: 13, 20 and 25

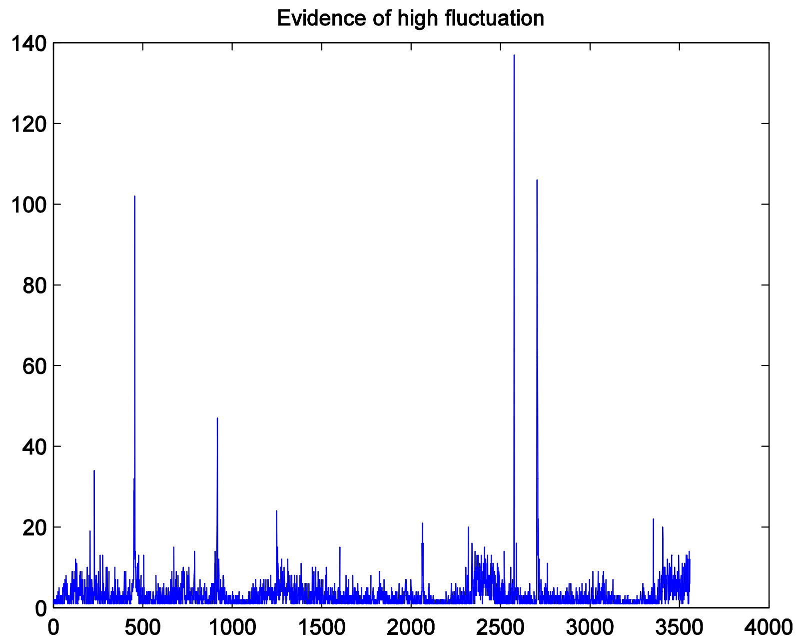


Fig.11 Weight of very high fluctuation of particles registered (time evolution). Ch25.

## Conclusions

Complex system methodology used in this study shows the particular effectiveness on the case where analytic or other consideration were not applicable or not mathematically appropriate. According to the candidate's/parties votes as by polling station reference on Albania (2001,2005,2009) the distribution is retrieved from a dynamic simulation on a very simple directed network using modified preferential attachment rule. We extended the idea on the simulation of possible corruption activities on first step that is non organized one. Here we reveal the effect of common negative opinions that do involve a considerable fraction of citizens on corruptive schema. Analyze of the extreme events on electoral results uncover the organizational behavior and vulnerable activities on strange deformation as wrong ballot box on 2007, the tabulation problems on 2005 and 2007 and tactical manipulation as the distribution is better fitted to a power law, whereas the invalid ballots are normally distributed indicating natural and causal voting errors. It is not possible to deduct this mechanism form other classic point of view highlighting the vast space of complex system methods application.

Using network and complex system approach we distinguish the differences on aerosol particles dynamics according to their radius and mass. Big particles found on aerosols and other pollution as measured on a station in Tirana are distributed as power law that is the number or fraction of particle registered within a minute fulfill a power law with negative exponent. It can be e result of self organization behavior or large variance caused from complexes and competitive phenomena.

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