IMPROVING THE CALCULATION METHOD OF QUARK-ANTIQUARK POTENTIAL FROM FERMIQCD

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Quantum Chromodynamics (QCD) is the theory of strong interactions. It is formulated in terms of quarks and gluons which are the basic degrees of freedom that make up hadronic matter. The approach to solve the QCD theory in non-perturbative regime, at the hadronic scale where coupling constant is large was introduced by Wilson (1974), named lattice QCD (LQCD). LQCD is QCD formulated on a discrete Euclidean space time grid. When the size of the lattice is taken infinitely large and its sites infinitely close to each other, the continuum QCD is recovered. An object of considerable theoretical interest in LQCD is the potential between a static quark and antiquark. In LQCD the static quark-antiquark potential can be calculated from the Wilson loops. This work makes use of results produced by the High-Performance Computing Infrastructure for South East Europe's Research Communities (HP-SEE). In this paper we use parallel computing with FermiQCD software to recalculate the static quark-antiquark potential improving thus our previous method. The improvement consists in symmetrization of Wilson loops, because in the Euclidian space-time, there is no specified direction for time or space. We also have improved the statistical errors of lattice spacing using weighted coefficients with Jackknife method. Using simulation with SU(3) gauge field configuration we have derived quark-antiquark potential for different values of coupling constant and for 8^4, 12^4 and 16^4 lattice volume. The calculations are performed for 100 configurations, statistically independent, of the gauge fields of the lattice. The behavior of calculated potential from quarks distance, shows that quarks are confined into hadrons. We present the results of lattice spacing for different lattice volume and the respective statistical errors. The calculated values of lattice spacing are compared with them from parameterization and results show that they are within the range of errors found.

***Keywords*:** *lattice spacing, quark-antiquark potential, SU(3) gauge theory.*

1. Introduction

Although Quantum Chromodynamic (QCD) is the best candidate for a theory of the strong interactions, the quantitative information that can be extracted is still limited. Analytic or perturbative solutions in low-energy QCD are impossible due to the highly nonlinear nature of the strong force. The lattice formulation, Lattice-QCD (Gattringer & Lang, 2009) is one of the most powerful non-perturbative methods. It is a lattice gauge theory formulated on a grid or lattice of points in space and time, where fields representing quarks are defined at lattice sites while the gluon fields are defined on the links connecting neighboring sites. This formulation of QCD in discrete rather than continuous space-time naturally introduces a momentum cut off at the order $1/a$, where a is the lattice spacing, which regularizes the theory. This approximation approaches continuum QCD as the spacing between lattice sites is reduced to zero. As a result lattice QCD is mathematically well-defined. Its power comes mainly from the possibility to evaluate physical quantities "exactly" via numerical simulations, yielding a method to test whether QCD provides the correct framework for describing strong interactions. The lattice spacing plays the role of the ultraviolet regulator, rendering the quantum field theory finite.

An object of considerable theoretical interest that has been studied for almost 30 years is the potential between a static quark and antiquark. It defines the energy of gauge fields in the presence of two static color sources separated by a distance R. At large distances quark confinement shows up and perturbative methods are no longer able to describe the behavior of physical observables; in this context, lattice gauge theories play an important role. Lattice results can be used to investigate non-perturbative features of the static quark potential as well as to test the range of validity of perturbation theory, which is still an important phenomenological issue which has to be investigated more precisely. The Creutz and Moriarty (1982) computer experiment to measure the static quark-antiquark potential in essence have been that quarks are *confined*, so that the static potential exhibits a linear rise at large distances. Recent lattice studies, (Schilling and Bali, 1993), (Deldar, 2000), (Bali, 2000), (Parisi, Petronzio and Rapuano, 1983), confirm the existence of the linear potential for the intermediate distances or the fundamental and higher representations in SU(3). Based on these measurements, quarks are confined in all representations of SU(3).

Motivated by these important physical results, in our previous work (that was presented on HP-SEE User Forum 2012, October 17-19, 2012, Belgrade, Serbia) we have calculate the static quark-antiquark potential in quenched approximation using high parallel calculation techniques for lattices with different volume. The results that we have taken were preliminary physic results and they needed improvement because the error of lattice spacing a was too small to justify the difference between $a\_$ *calculated* and a from parameterization. In this paper we present our latest results improving our method of calculation of interquark potential, by symmetrizing Wilson loops, because in the Euclidian space-time, there is no specified direction for time or space. We also have improved the statistical errors of lattice spacing using weight coefficients with Jackknife method.

2. Quark-antiquark potential from Wilson loops

Wilson loops were introduced by Wilson (1974**)** as an attempt to a non-perturbative formulation of quantum chromodynamics, or at least as a convenient collection of variables for dealing with the strongly-interacting regime of QCD. The measurements on the lattice of the potential, related to Wilson loop measures, have parallel histories initiated in (Stack, 1983), (Grifiths, Michael and Rakow, 1983), (Campbell, Michael and Rakow, 1984), (Campbell, Huntley and Michael, 1988), (Perantonis, Huntley and Michael, 1989).

We have used the standard methods to compute the quark-antiquark potential. The quark-antiquark potential can be extracted by large time behavior of the Wilson loops. We start from a closed rectangular path C(R,T) with extension *R x T*. The potential is extracted from so called Wilson loops W(R,T), which are defined to be the trace of path-ordered products of link variables $U_\mu(n)$ along the path C (R,T). This loop construction corresponds to the world lines of a quark-antiquark pair at rest, separated by distance R from each other and "travelling" over time separation T. In Euclidian time, this observable will reveal the static "*ground state"* energy for large T values. To calculate the energy we start from the correlate function of quark antiquark operator for different time:

$$W(R,T) = \langle 0|O_{\bar{R}}(0)O_R(T)^*|0\rangle = \langle O_{\bar{R}}(0)O_R(T)^*\rangle \tag{1}$$

when operator $O_{\bar{R}}(T)$ is gauge invariant and is defined as:

$$O_R(T) = \bar{q}(T,0)U\big((T,0)\to(T,R)\big)q(T,R) \tag{2}$$

and $U\big((T,0)\to(T,R)\big)$ is gauge field that connect static quarks from point (T,0) to point (T, R). Finally the Wilson loops can be defined as:

$$W(R,T) = \langle tr\, U\big((0,0)\to(0,R)\big)U\big((0,R)\to(T,R)\big)U\big((T,0)\to(T,R)\big)^* U\big((0,0)\to(T,0)\big)^*\rangle \tag{3}$$

The Wilson loops can be written as:

$$W(R,T) = \sum_{n>1} c_n e^{-V_n(R)T} \tag{4}$$

Where the ground state is $V_1(R) \equiv V(R)$ and the other values, for n>1, are the potentials of the excited states. So we can extract potential assuming:

$$W(R,T) \cong c_1 e^{-V(R)T}, \tag{5}$$

and than calculate the effective potentials from:

$$V(R)_{eff} = -\log\frac{W(R,T+1)}{W(R,T)} \tag{6}$$

Finally, for different R we can fit the effective potential according to theoretical model in physical unit:

$$V(R)_{eff} = V_0 + KR + \frac{\alpha}{R} \tag{7}$$

where α is a constant (the coefficient of the Coulomb-like term) and *K* the string tension parameter. We can take the representation of this model in lattice unit by multiplying with lattice spacing the equation (7):

$$\bar{V}(R)_{eff} = \bar{V}_0 + \bar{K}\frac{R}{a} + \frac{a}{R}\bar{\alpha}. \tag{8}$$

Monte Carlo calculations of lattice QCD in the quenched approximation have reached considerable precision. For example, the accuracy of hadron mass calculations using the Wilson action is quoted to be well below 1% at finite values of the lattice spacing (Aoki, 1998)**,** and the running of the coupling is known to a precision of around 1% over energy scales varying by two orders of magnitude (Luscher, Sommer, Weisz and Wolff, 1994**)**, (Capitani, 1998). In comparison to such a level of accuracy, a low energy reference scale in the Yang-Mills theory (Hooft, 2005) is known with much worse precision, despite the fact that such a scale is very important for the analysis of the results as is shown by Edwards, Heller and Klassen (1997). For such purposes was introduced a reference scale, r_0 (Sommer, 1994). This length scale is defined in terms of the force, F(R), between external static charges in the fundamental representation. It is the solution of $r_0^2 F(r_0) = 1.65$. The constant on the right side was chosen such that r_0 has a value of approximately 0.5 fm in QCD. We have calculated lattice spacing *a* according to Guagnelli, Sommer and Wittig (1998). The physical volume of the lattice (L^4) with length L is related to the lattice volume (N^4) with N point for direction by L=aN. In order to take physical length L of the lattice constant we changed the value of coupling constant $\beta = 6/g^2$ (g is the QCD coupling) in each simulation with different lattice volume (8^4, 12^4, 16^4). The changed values of β are taken from parameterization of the low-energy reference scale r_0 according to Guagnelli, Sommer and Wittig (1998).

3. Parallel computations with FermiQCD

LQCD is based on complex numerical algorithms derived from a mathematical analogy between "Feynman paths" in Quantum Mechanics and "steps" in the Markov Chain Monte Carlo algorithm. LQCD is a large scale computing project where computations are typically very expensive and run on dedicated supercomputers and large computer clusters for many months. Numerical lattice QCD calculations using Monte Carlo methods can be extremely computationally intensive, requiring the use of the largest available supercomputers. In this paper we use a special software called Fermiqcd, (Di Pierro, 2001) (Di Pierro, 2002), (Di Pierro, 2004), which is a collection of classes, functions and parallel algorithms for lattice QCD written in C++. It's a library that includes C++ methods for matrix manipulation, advanced statistical analysis (such as Jackknife and Boostrap) and optimized algorithms for interprocess communications of distributed lattices and fields. These communications are implemented using Message Passing Interface (MPI) but MPI calls are *hidden* by the high level algorithms that constitute Fermiqcd. Fermiqcd works also on single processor computers and, in this case, MPI is not required. One of the main differences between Fermiqcd and libraries developed by other collaborations is that it follows an *object oriented design* as opposed to a procedural design. Fermiqcd should not be identified exclusively with the implementation of the algorithms but also with the strict specifications that define its Application Program Interface. The objects of the language include complex numbers (*mdp complex*), matrices (*mdp matrix*), lattices (*mdp lattice*), fields (gauge field, fermi field, staggered field), propagators (fermi propagator) and actions.

The algorithm of the code that we have written in Fermiqcd follows this steps below:

Step 1: Include Fermiqcd libraries

Step 2: Start (*mdp.open_wormholes(argc,argv*))

Step 3: Define this parameters:

- Lattice volume
- SU(n) gauge group
- Number of gauge configurations or number of Monte Carlo steps

- Coupling constant (beta) that sets lattice spacing

Step 4: make:

- A 4D lattice (*mdp_lattice lattice(4,L))*
- A gauge field U (*gauge_field U(lattice,n)*)
- A random gauge configuration (*set_hot(U)*)

Step 5: Loop over the Monte Carlo steps or over number of configuration

Step 6: Make a generic path on lattice to construct the shape of Wilson loop

Step 7: Loop over all possible paths in $\mu\nu$ plane, for $\mu = 0,1,2,3$ and $\nu = 0,1,2,3$

Step 8: Calculate the real part of the trace of path-ordered products of link variables $U_\mu(n)$ along the path, so the Wilson loops

Step 9: Save the Wilson loops in a *.dat* file format.

Step 10: Close with (*mdp.close_wormholes())*

4. Results

The simulations are made with Wilson action on 8^4, 12^4, 16^4 lattices at three lattice spacing for 100 SU(3) gauge field configurations and carried out on the BG-HPC cluster that is located in Bulgaria (part of HP-SEE project), using Fermiqcd software.

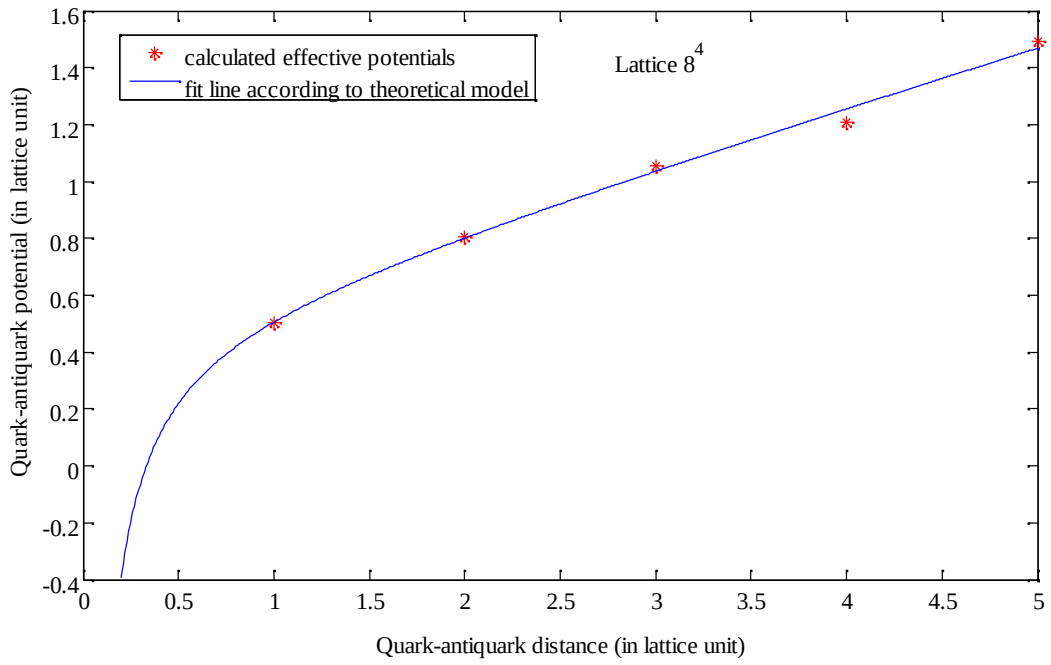

Figure 1 – Graph of quark-antiquark potential for beta=5.7, 8^4 lattice volume, in lattice unit

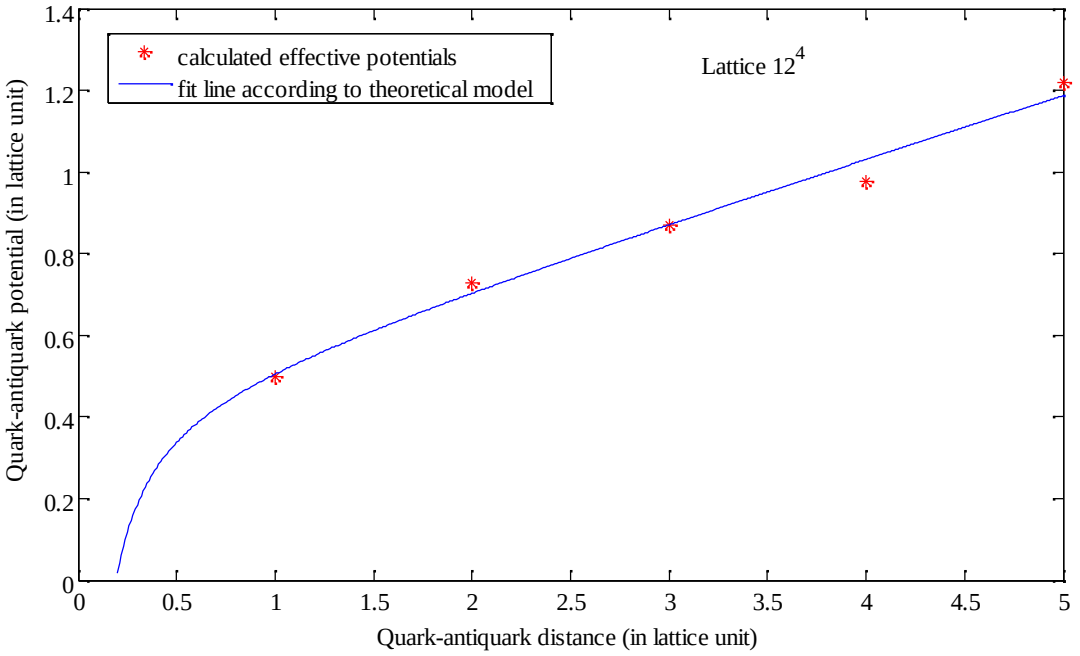

Figure 2 - Graph of quark-antiquark potential for beta=5.85, 12^4 lattice volume, in lattice unit

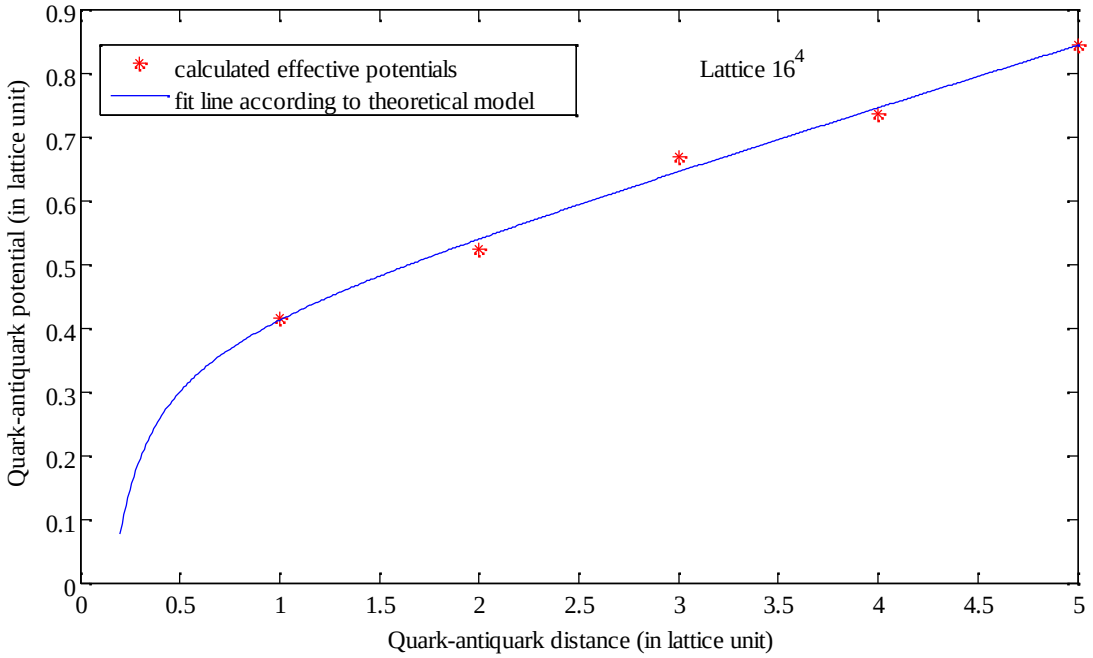

Figure 3 - Graph of quark-antiquark potential for beta=6, 16^4 lattice volume, in lattice unit

As we can see from the behavior of the potential from quark distance, for different lattice volume, Fig.1, Fig.2, Fig.3, the quark-antiquark potential is dominated by a Coulomb–

like term at short distances with an *R* dependent coupling $V(R) \cong V_0 + \alpha/R$ for *R* small and by a linearly rising term at large distances $V(R) \cong V_0 + KR$ for *R* large, that confirm the fact of *confinement* of quarks within hadrons as in Schilling (1993), Deldar (2000), Bali (2000), Parisi, Petronzio and Rapuano (1983). The recalculation of statistical errors for the coefficients of the model is made using Jackknife method (Miller, 1974), but including weight coefficients of errors of the potential.
The results are presented in Table 1.

Table 1: The values of lattice spacing *a* for different lattice volume with respective statistical errors

Lattice volume	Coupling constant	Lattice spacing *a* from parameterization (in *fm*)	Calculated lattice spacing *a* (in *fm*)	Statistical error of *a*
8^4	5.7	0.1707	0.1735	0.0145(76)
12^4	5.85	0.1230	0.1273	0.0214 (38)
16^4	6	0.0931	0.0973	0.0594(74)

As we can see from Table 1, the values of lattice spacing for different lattice volume are calculated within the range of statistical error. Our objective in this paper was to improve our previous results of the static quark-antiquark potential and lattice spacing errors.

Conclusions: The parallel computation techniques with Fermiqcd will be the most important feature in our future work. We tested the standard method of computing quark-antiquark potential from planar Wilson loops using parallel computation techniques with Fermiqcd. We found that the quark-antiquark potential is dominated by a Coulomb–like term at short distances for small *R* and by a linearly rising term for large *R*. This behavior of quarks potential confirms an important properties of QCD in low energy regimes, the so called *quark confinement*. The calculated values of lattice spacing compared with them from parameterization show that they are within the range of errors found. These values of lattice spacing can be used to set the scale of the theory, so with them we can convert quantity from lattice unit in physical unit.

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