CAUSAL MODELING IN MATHEMATICAL EDUCATION

Božidar Tepeš[1], Gordana Lešin[2], Ana Hrkač[3]

[1]Faculty of Teacher Education, University of Zagreb, Savska cesta 77, bozidar.tepes@ufzg.hr
[2] Kindergarten M. Sachs, Sachsova 5, Zagreb, g.lesin.zg@gmail.com
[3] Kindergarten M. Sachs, Sachsova 5, Zagreb, analov@gmail.com

Abstract

In this paper, the authors define mathematical knowledge for kindergarten children. The basic objective was to measure the mathematical knowledge, skills and abilities or mathematical competences in mathematics education in kindergartens. The fundamental mathematical concepts were divided into three areas of mathematics: geometry, arithmetic and data. Paper describes the statistical set of 83 children, 65 to 85 months of age, from Milan Sachs kindergarten from Zagreb. The authors describe the variables measuring math competences. Six measuring variables were described for the geometry, and seven measuring variables for the arithmetic. Measuring variables are tasks which children solved, together with the evaluated results of the tasks. Structural models for geometry and arithmetic were determined by statistical analysis of the partial correlation coefficients. Further statistical analysis defined causal models of geometry and arithmetic. Causal models are presented with acyclic graphs. From causal model geometry, underlying concepts are circle and rectangle. The last concept is left – right relationship, associated with a high degree of body movement coordination (touch the left ear with the right hand). Underlying competencies relating to understanding numbers up to 10 and adding with the number 1 are from arithmetic's causal models. Last concepts of causal model were counting to 30 and adding with the number 2. Those concepts require a high degree of concentration for a long period of time, together with the high degree of mathematics abstraction. For the purpose of further research, it is necessary to increase the statistical set or the number of children examined. Test materials must be standardized and must allow for higher gradation of results. Future studies should include part of mathematical competence relating to the numerical data that have descriptive characteristics expressed by words and letters. This is because most children are taught letters in kindergarten, before enrolling in primary school.

***Keywords*:** mathematical knowledge, mathematical skills, mathematical abilities, causal model

1. Introduction

At the beginning, mathematical knowledge, skills and abilities or mathematical competence in pre-school education on the basis of the Draft framework mathematics curriculum are defined [1]. For the successful studying of mathematical competence, it is necessary to examine the causal connection between the parts of mathematical competence. The paper uses two causal models [2]. The first is a causal model for geometry and the second is causal model for arithmetic. Models are determined from the statistics group of 83 children, 65 to 85 months of age, from kindergarten Milan Sachs from Zagreb. Six measuring variables were described for the causal model for geometry, and seven measuring variables for the causal model for arithmetic.

2. Mathematical competence in pre-school education

When defining mathematical competence in preschool education, one must start from the goals. Main goal is the introduction to the basic mathematical concepts that will be used and studied by preschool children in the elementary and secondary school. [1].

Fundamental mathematical concepts can be divided into three areas of mathematics:

- Geometry
- Arithmetic
- Data

Each of these areas has its own mathematical competence.

Mathematical competence for geometry includes:

- Space
  - Relations more and less
  - Relations before and after
  - Route above and below
  - Relation between left and right
- Geometric objects
  - Distinguishing lines and surfaces
  - Straight and curved lines and surfaces (lines and planes)
  - Recognition of simple shapes (triangle, square and circle)
  - Recognition of simple bodies (pyramid, cube and sphere)

Mathematical competence in arithmetic includes:

- Numbers
  - Recognizing numbers,
  - Sequencing numbers,
  - Writing numbers
- Relations and operations with numbers
  - Comparing numbers,
  - Adding numbers,
  - Subtracting numbers

Mathematical competence of the data includes:

- Data

Collecting data,
Sorting data,
Creating data table

Data display

Drawing data,
Distinguishing colors,
Drawing data on the computer

The goal of our work is to measure mathematical competence of geometry and arithmetic in kindergarten education. It is therefore necessary to determine the statistical set and features of the statistical set that is being measured. Selected features will be measuring variables for our causal models.

3. Statistical set and measuring variable

Elements of statistical set were 83 children from the Milan Sachs kindergarten [3]. Testing was performed as a part of an ordinary testing of the children's readiness for school and development monitoring, which is part of the preschool education's curriculum. [4] The age structure of children examined is shown in Table 1:

Table 1 Children's ages

Children's ages (months)	Number of children
(65) – 70	28
70 – 75	15
75 – 80	21
80 – (85)	19
Total	**83**

Measuring variables for causal model geometry were:

1. Left and right relationship (L-R)
2. In the front and behind relationship (F-B)
3. Above and below relationship (A-B)
4. Recognizing triangle (TRI)
5. Recognizing rectangle (REC)
6. Recognizing circle (CIR)

Measuring variables for causal model of arithmetic were:

1. Counting to 30 (C30)
2. Understanding the numbers to 10 (S10)
3. Knowing the number of fingers on both hands (NFH)
4. Adding +1 (A +1)
5. Adding +2 (A +2)
6. Subtracting -1 (S-1)
7. Subtracting -2 (S-2)

Figure 1 and Figure 2 represents some instruments for measuring variables of the front and behind relationship (F-B) and the above and below relationship (A-B).

Figure 1 Relation in front of and behind   Figure 2 Relation above and below

Each measurement variable has a particular task which will be described.

Tasks for measuring variables of geometry are:

1. Left - right relationship (L-R)

Knowledge of the left - right relationship was examined within orientation to one's own body. The child was asked to show, for example, his right leg, left ear, left eye and other body parts. If the child knew relationship he was evaluated with 1, and if the child didn't know he was evaluated with 0.

2. In front of and behind relationship (F-B)

With the help of pictures (Figure 1) the child was asked: Who is in front of the hen? Who is behind the girl? If the child knew relationship he was evaluated with 1, and if the child didn't know relationship he was evaluated with 0.

3. Above and below relationship (A-B)

With the help of the table and train toy, child was asked the following question: What is above the toy train? What's under the table? (Figure 2) If the child knew relationship he was evaluated with 1, and if the child didn't know relationship he was evaluated with 0.

4. Recognizing triangle (TRI)

The child was shown pictures of various geometric shapes and asked: Where is triangle? If a child recognized triangles he was evaluated with 1, and if the child didn't recognize he was evaluated with 0.

5. Recognizing rectangle (REC)

The child was shown pictures of various geometric shapes and asked: Where is rectangle? If the child recognized rectangles he was evaluated with 1, and if the child didn't recognize he was evaluated with 0.

6. Recognizing circle (CIR)

The child was shown pictures of various geometric shapes and asked: Where is the circle? If the child recognized circles he was evaluated with 1, and if the child didn't recognize he was evaluated with 0.

Tasks for measuring variables on arithmetic are:

1. Counting to 30 (C30)

The child was asked to count. Expected knowledge is counting to 30. The child that counted to 30 was evaluated with 1, the child that counted between numbers 10 and 20 was evaluated with 2, and the child that counted only up to 10 or less was evaluated with 0.

2. Understanding the numbers to 10 (S10)

More than 10 crayons were placed on the table. The child was asked to count them, joining one number and one crayon, with the proper sequence of numbers. If the child understood counting to 10 he was evaluated with 1, and if the child didn't understand he was evaluated with 0.

3. Knowing the number of fingers on both hands (NFH)

The child was asked: How many fingers do you have on both hands? The task allowed for counting of fingers. If the child successfully solved the task he was evaluated with 1, and if the child didn’t solve the task he was evaluated with 0.

4. Adding +1 (A +1)

Understanding of the operation of adding from 1 to 5 was examined by using the questions such as: You have 3 marbles and mother gives you 1 marble. How many marbles have you got? If a child successfully solved the task he was evaluated with 1, and if the child didn’t solve the task he evaluated with 0.

5. Adding +2 (A +2)

Understanding of the operation of adding from 1 to 5 was examined by using the questions such as: You have 3 marbles and mother adds 2 marbles. How many marbles have you got? If a child successfully solved the task he was evaluated with 1, and if the child didn’t solve the task he was evaluated with 0.

6. Subtracting -1 (S-1)

Understanding of the operation of subtracting from 1 to 5 was examined by using the questions such as: You have 3 marbles and your mother takes away 1 marble. How many marbles have you got? If a child successfully solved the task he was evaluated with 1, and if the child didn’t solve the task he was evaluated with 0.

7. Subtracting (S-2)

Understanding the operation of subtracting from 1 to 5 was examined by using the questions such as: You have 3 marbles and your mother takes away 2 marbles. How many marbles have you got? If a child successfully solved the task he was evaluated with 1, and if the child didn’t solve the task he was evaluated with 0.

4. Structural models of geometry and arithmetic

The structural model shows the correlation between two measuring variables, which is measured by the degree of statistical independence of two variables with knowing of other measuring variables [5]. The structural model is a graph in which the graph vertices are measurement variables, and the edges show correlation between measurement variables. Correlations are measured by the partial correlation coefficients $°_{i\,j|V\setminus\{i,\,j\}}$ of the vertices i and j knowing all other vertices $V\setminus\{i,\ j\}$ and set of all the vertices of the graph or measuring variables on models $V=\{1,2,...,n\}$. By defining limits value £ we determine the edges of the structural model. If $\left|°_{i\,j|V\setminus\{i,\,j\}}\right| >$ £ the vertices i and j are connected and that is the graph edge i —— j.

In the case of structural model of geometry partial correlation coefficients are shown in Table 2:

Table 2 The partial correlation coefficients $°_{i\,j|V\setminus\{i,\,j\}}$ of geometry

$i\setminus j$	F-B	A-B	TRI	REC	CIR
L-R	**0,318**	-0,134	0,112	**0,177**	0,097
F-B		**0,481**	0,057	-0,134	0,091
A-B			**0,326**	**0,165**	**0,206**
TRI				**0,182**	-0,152
REC					-0,083

If we define the limit value £ $=0,16$. Structural model geometry is graph on Figure 3.

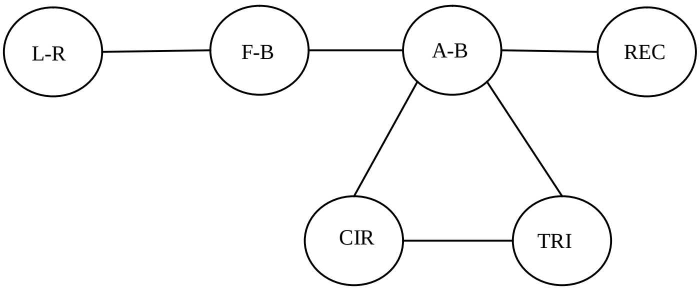

Figure 3. Structural model geometry

In the case of structural model of arithmetic, the partial correlation coefficients are shown on Table 3:

Table 3 The partial correlation coefficients $\rho_{ij|V\setminus\{i,j\}}$ of arithmetic

i \ *j*	S10	NFH	A+1	A+2	S-1	S-2
C30	-0,137	**-0,365**	0,066	0,174	0,102	-0,047
S10		0,120	**0,235**	0,092	**-0,200**	0,184
NFH			-0,049	0,089	**0,185**	-0,052
A+1				**0,413**	**0,349**	-0,158
A+2					-0,086	**0,253**
S-1						**0,819**

If we define the limit value $\varepsilon = 0{,}185$. Structural model arithmetic is graph on Figure 3

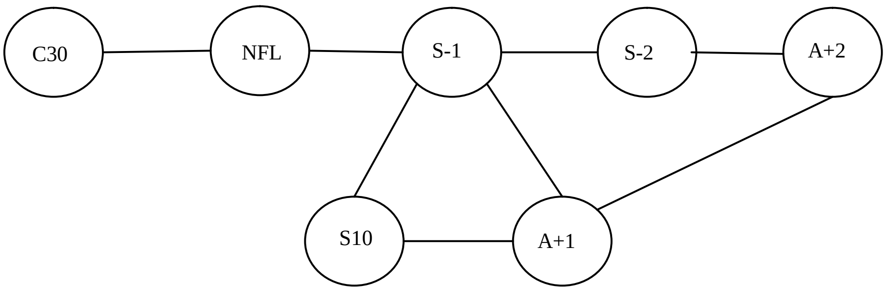

Figure 4. Structural model arithmetic

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5. Causal models of geometry and arithmetic

From the structural models, causal model is determined by further analysis of partial correlation coefficients using an algorithm J. Pearl [6] which was developed in the work of JP

Pellet & A. Elisseff [7]. The idea of the algorithm is to break the triangular structures in the structural graphs and search V-structures that have a special meaning in the causal models. The triangular structure in the structural model of geometry is $(A-B) — CIR — TRI — (A-B)$. Once the triangular structure is destroyed, new V-structure is $CRC \rightarrow TRI \leftarrow (A-B)$. Respecting that the causal graph is acyclic graph and independent, we come to the causal model of geometry in Figure 5.

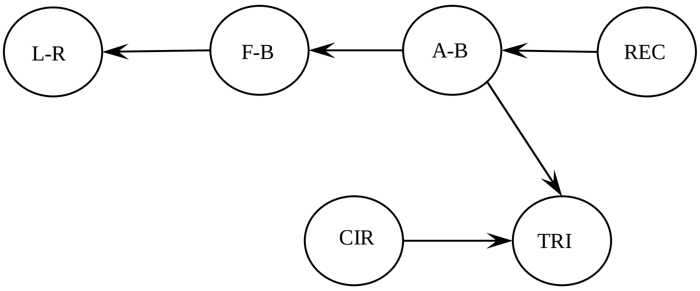

Figure 5. Causal model geometry

The triangular structure in the structural model of arithmetic is $S10 — (S-1) — (A+1) — S10$. Once the triangular structure is destroyed, new V-structure is $S10 \rightarrow (S-1) \leftarrow (A+1)$. Respecting that the causal graph is acyclic graph and independence, we come to the causal model of arithmetic in Figure 6.

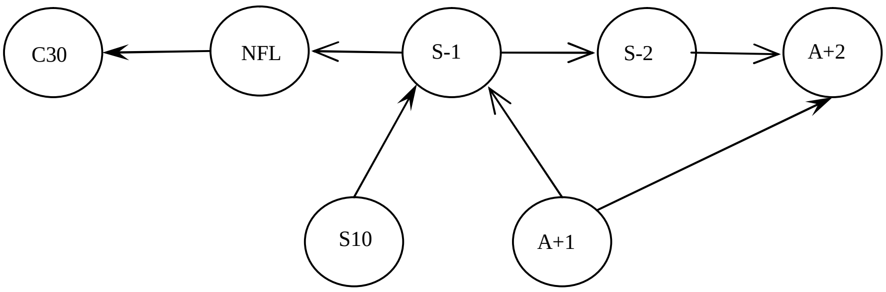

Figure 6. Causal model arithmetic

From the causal model of geometry (Figure 5), we can conclude that circle (CIR) and rectangle (REC) are two basic concepts upon which the kindergarten child adopts geometric competences. The first relation a child should master is above and below (A-B). The last relation a child should master is left and right relationship (LR) because it is associated with examining of the body parts and requires a high degree of coordination of movements of the body (touch the left ear with the right hand).

From causal models of arithmetic (Figure 6), we can conclude that understanding of numbers up to 10 (S10), and adding +1 (A +1) are two fundamental concepts in arithmetic competencies. The last operation the child should master is adding +2 (A +2) and counting to 30 (C30). Counting to 30 and adding with number 2 requires a high degree of concentration for long period of time, together with a high level of abstraction.

6. Conclusion

This work demonstrated the causal structure of mathematical competences in preschool education. For the purposes of adopting mathematical competences, causal model refers to the order of adopting of mathematical concepts. For the purpose of further research, it is necessary to increase the statistical set or the number of children examined. Test materials must be standardized and must allow for higher gradation of results. The study should include part of mathematical competence relating to the data [8] that have both numeric and descriptive characteristics expressed by words and letters. This is because most children are taught letters before enrolling in primary school. The curriculum for children in kindergarten in Croatia is based on social relations, and should have the educational structure of mathematical competences accustomed to the age and level of knowledge that children acquire by using information and communication technologies of contemporary society [9].

7. References

[1] Hrvatsko matematičko društvo (2009), Prijedlog okvirnog matematičkog kurikula (dorađena inačica), http://www.matematika.hr/_news/9912/Matematika-NOK-15102009-po_domenama.pdf

[2] Tepeš, B. (2009), Statistički modeli na grafovima, Učiteljski fakultet, Zagreb, http://www.ufzg.unizg.hr/wp-content/uploads/2012/02/Statisti%C4%8Dki-modeli-na-grafovima1.pdf

[3] Dječji vrtić Milana Sachsa (2011), Zagreb , http://www.dvmilanasachsa.hr/

[4] Nacionalni okvirni kurikulum za predškolski odgoj i obrazovanje te opće obvezno i srednjoškolsko obrazovanje (2011),Ministarstvo znanosti, obrazovanja i športa Republike Hrvatske, http://public.mzos.hr/Default.aspx?sec=2685

[5] J. Whittaker (1989), Graphical models in applied multivariate statistics, John Wiley & Sons

[6] Pearl J..Causality: Models (2000), Resoning, and Inferece, Cambridge University Press, 2000

[7] Pellet, JP. & Elisseeff, A.(2007), A Partial Correlation-Based Algorithm for Causal Structure Discovery with Continuous Variables. in M. R. Berthold, J. Shawe-Taylor and N. Lavrač (eds), *Advances in Intelligent Data Analysis VII, 7th International Symposium on Intelligent Data Analysis*, pp. 229–239.

[8] Kurikulum za predškolski odgoj, Lpfő 98 (2001), Ministarstvo obrazovanja i znanosti Švedske i nacionalna agencija za obrazovanje, http://www.see-educoop.net/education_in/pdf/swed_presch_curric_eng-oth-hrv-t06.pdf

[9] Kliman, M., Russel, S. J., Wright T., Mokros J. (2006) Mathematical Thinking at Grade 1., Technical Education Research Centers& Scott Foresmann, Cambridge MA